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Shortcuts to adiabaticity in simple classical systems

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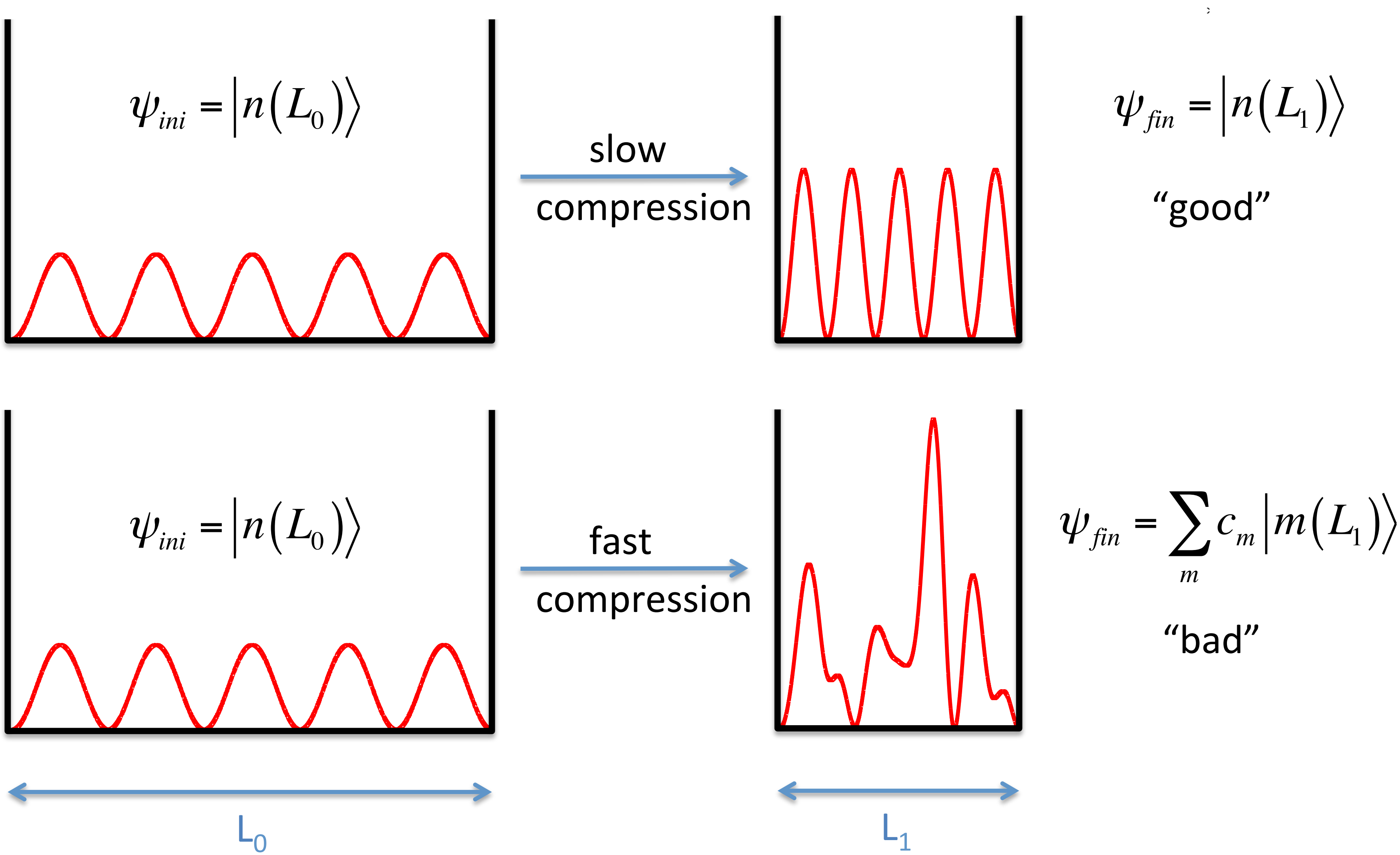
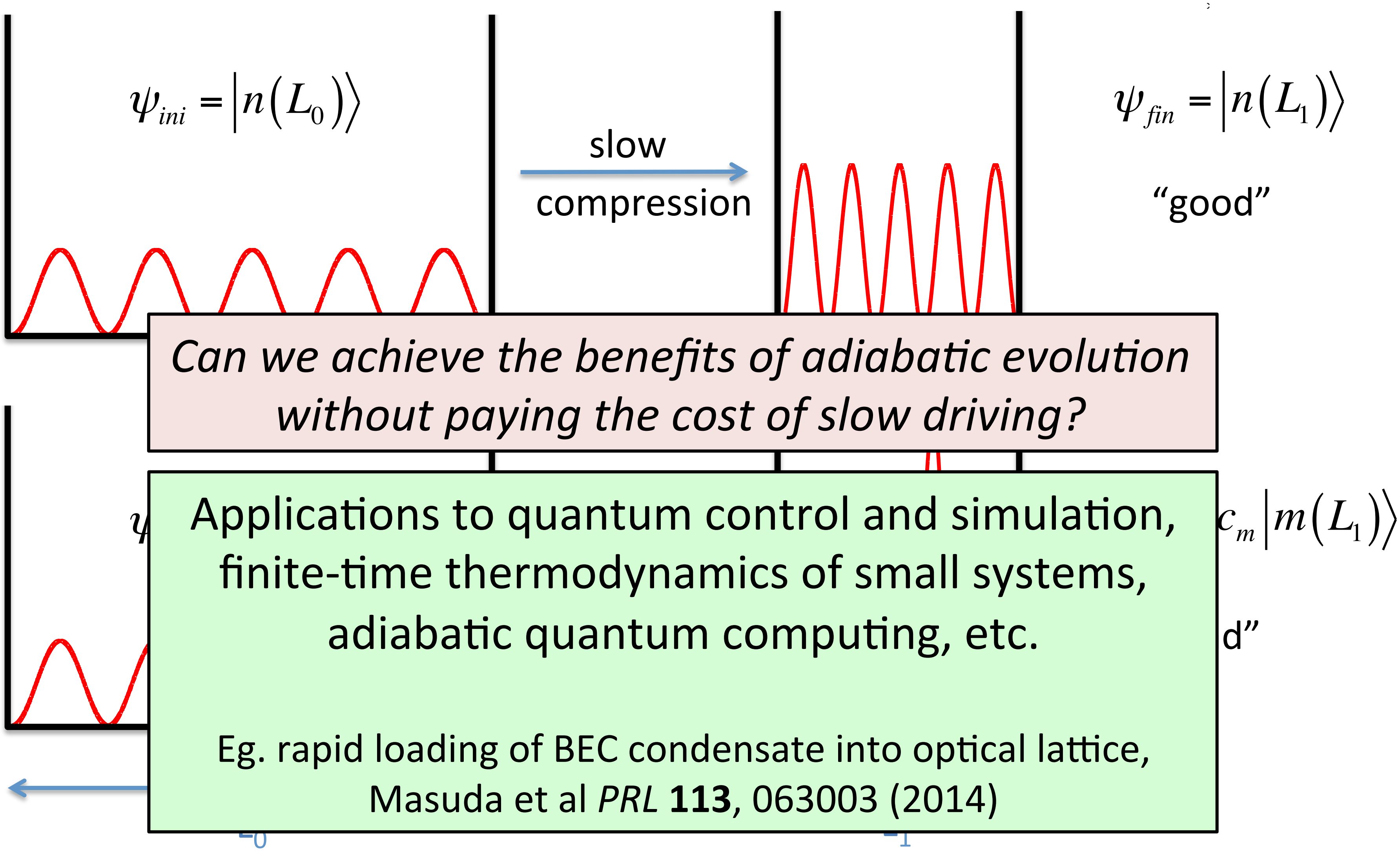
Quantum adiabatic theorem



Quantum adiabatic theorem



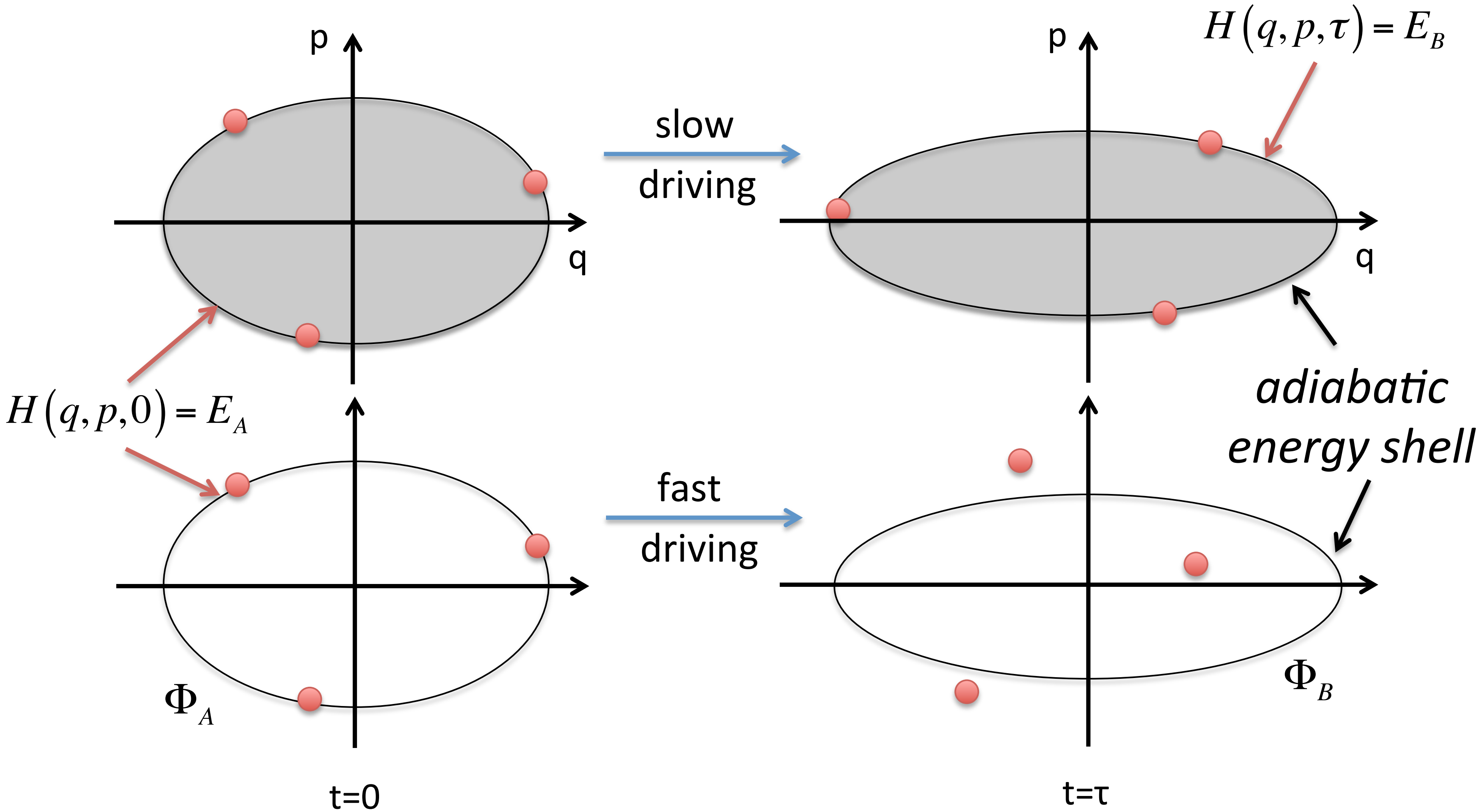
Can we achieve the benefits of adiabatic evolution without paying the cost of slow driving?

Applications to quantum control and simulation, finite-time thermodynamics of small systems, adiabatic quantum computing, etc.

Eg. rapid loading of BEC condensate into optical lattice, Masuda et al *PRL* **113**, 063003 (2014)

Classical adiabatic theorem

1 degree of freedom , H(q,p,t) = Hamiltonian of interest

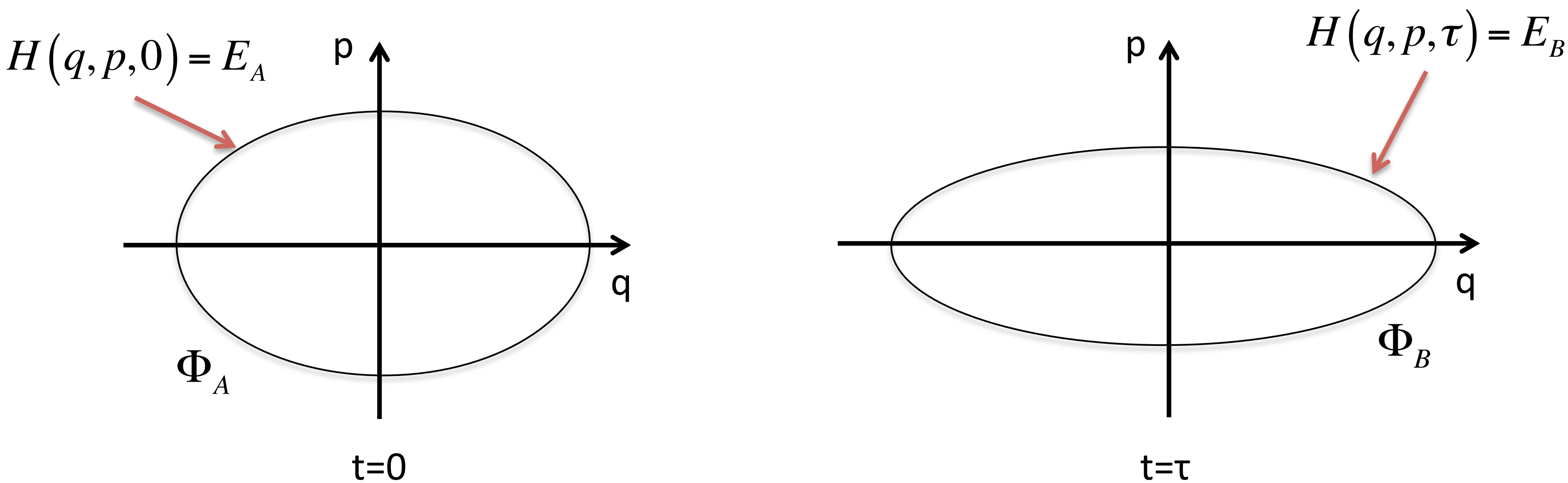


Statement of the problem

Given: $H(q,p,t)=\dfrac{p^2}{2m}+V(q,t) \qquad 0\le t\le\tau$ + initial energy E_A

Assume: $\dfrac{\partial H}{\partial t}=0 \quad at\ t=0\ \&\ t=\tau$ H begins & end "at rest"

the adiabatic energy shell is a simple closed curve at all times



Statement of the problem

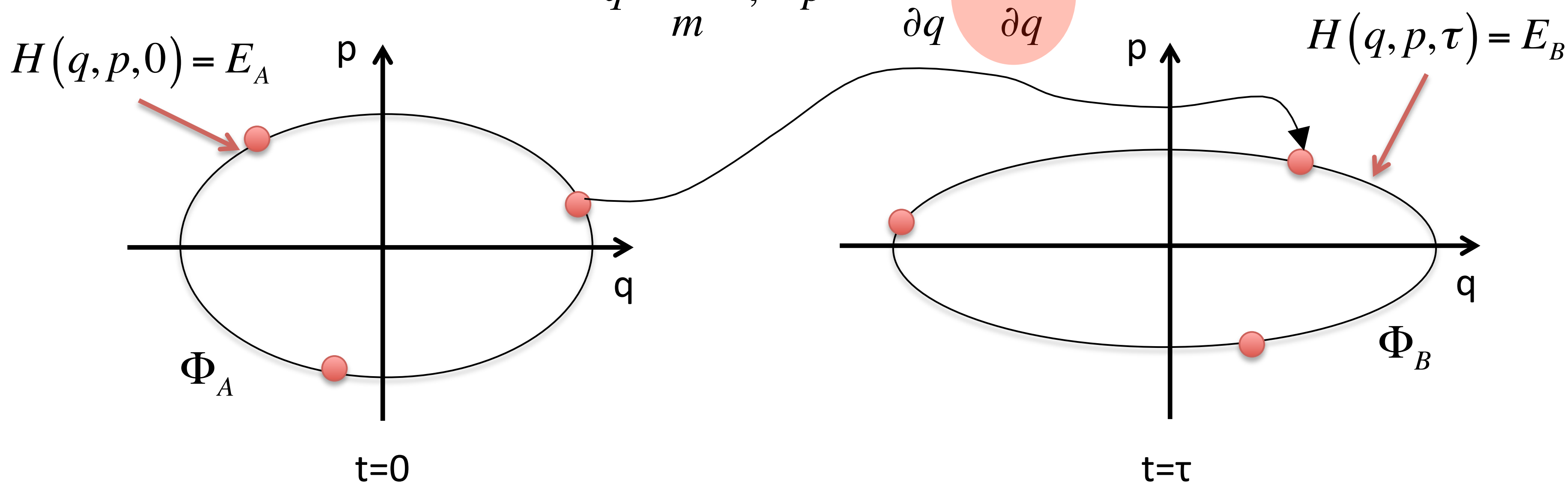
<u>Given</u>: $H(q,p,t) = \dfrac{p^2}{2m} + V(q,t) \qquad 0 \le t \le \tau$ + initial energy E_A

Find U(q,t) such that

Hamiltonian evolution under H(q,p,t) + U(q,t) maps Φ_A to Φ_B .

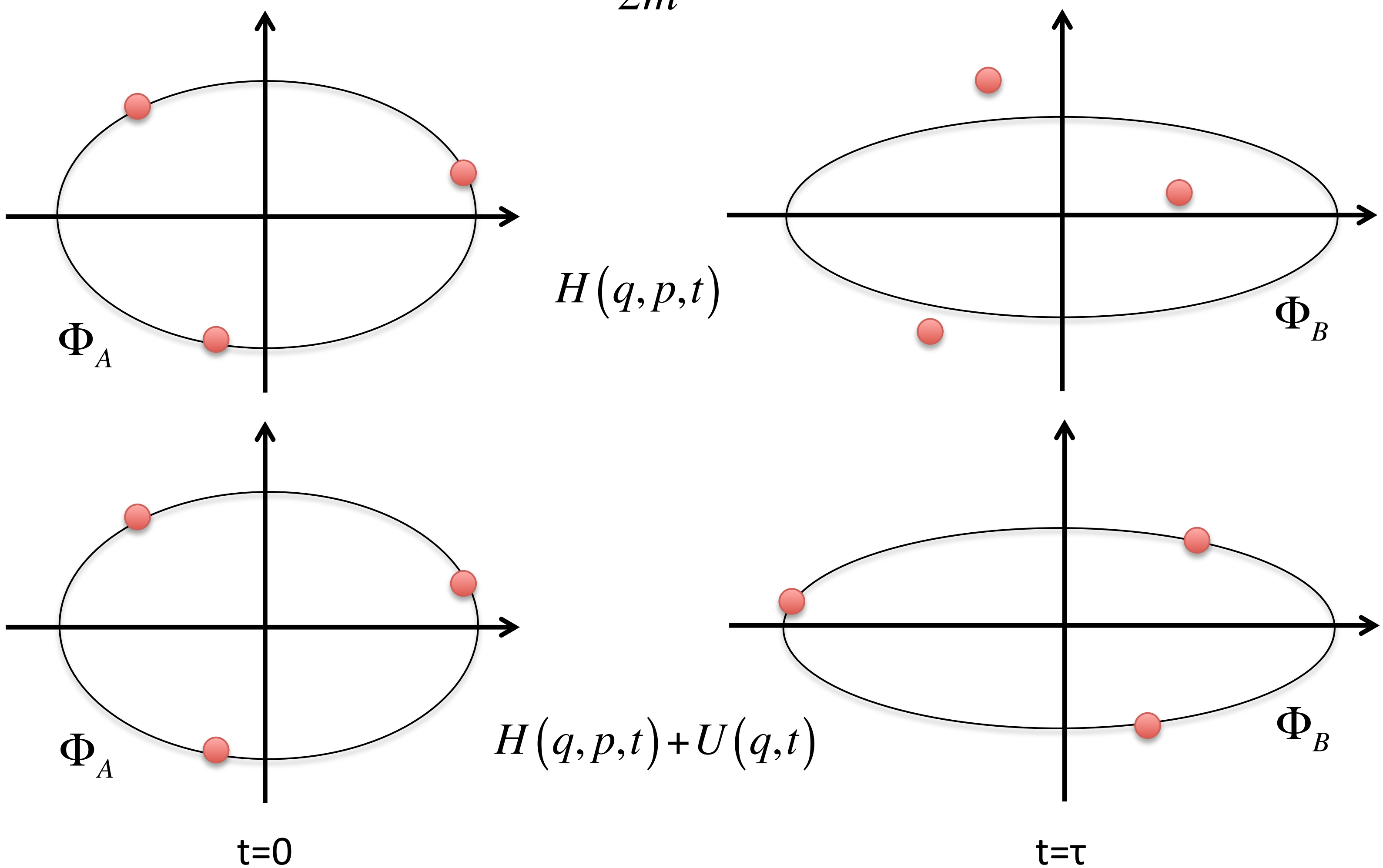
counter-diabatic

$$\dot{q} = \frac{p}{m} \quad , \quad \dot{p} = -\frac{\partial V}{\partial q} - \frac{\partial U}{\partial q}$$

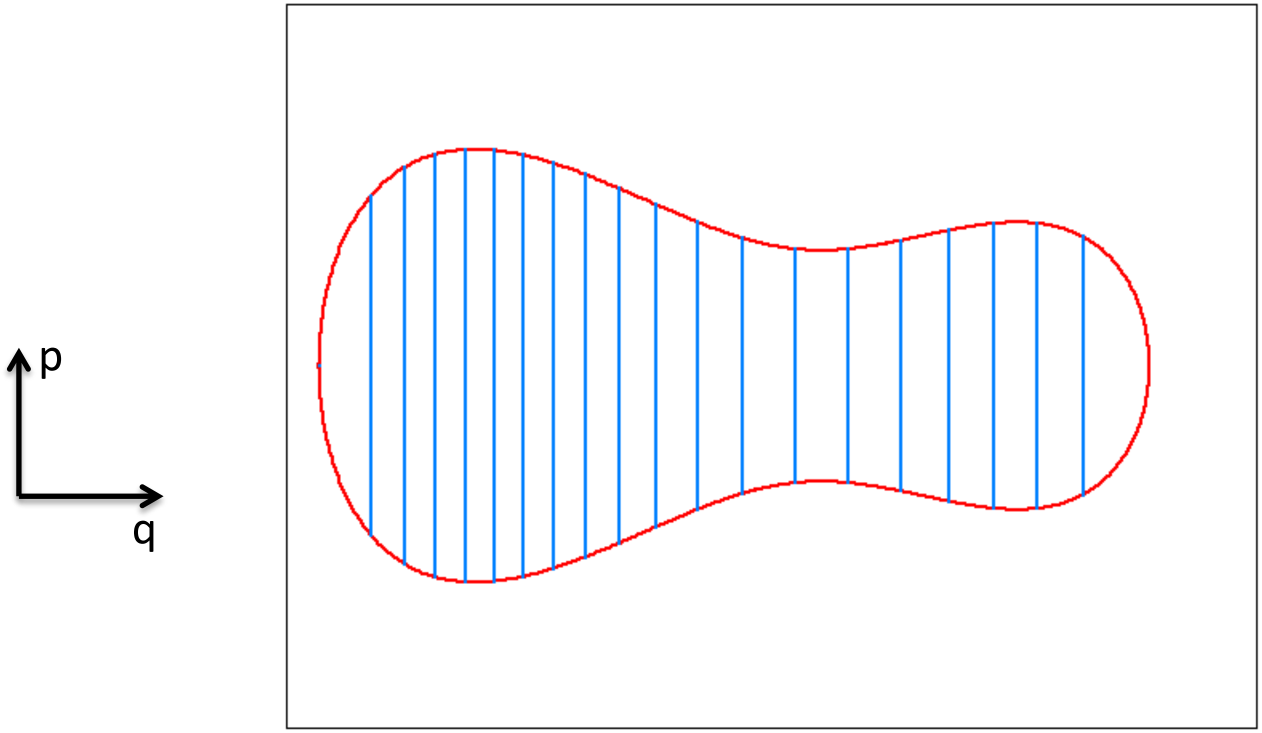


Statement of the problem

$$H(q,p,t) = \frac{p^2}{2m} + V(q,t)$$



Construction of U(q,t)   [given H(q,p,t) , E_A]

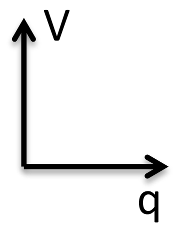


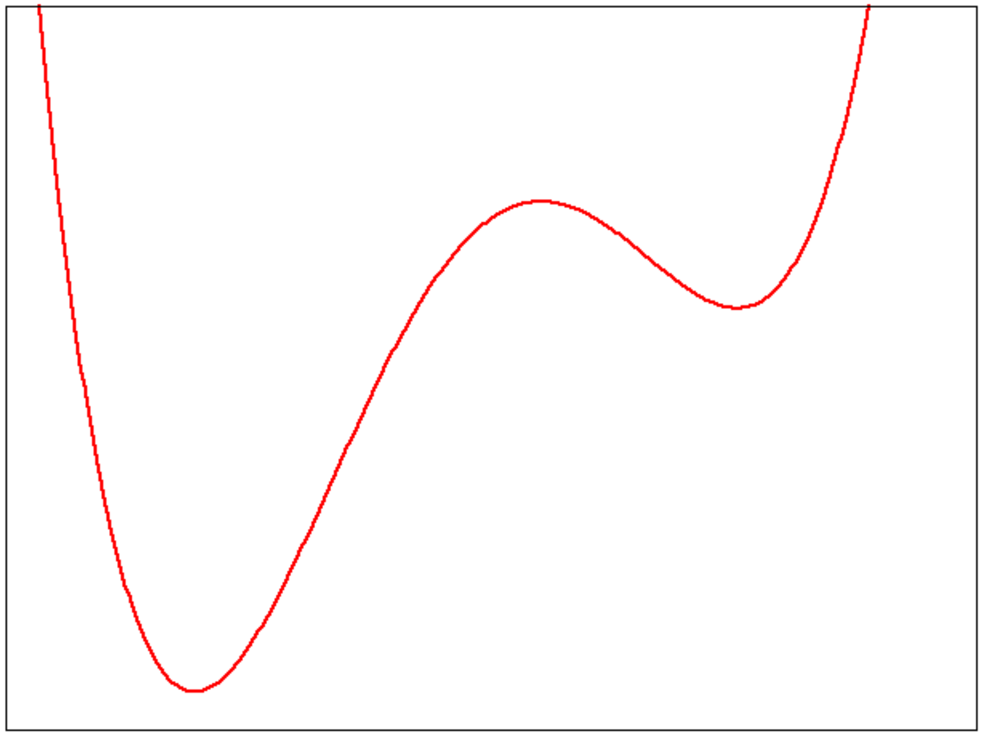
- vertical lines at $q_1(t)$, $q_2(t)$ … $q_{N-1}(t)$ divide the adiabatic energy shell into N strips of equal phase space volume (N -> ∞)
- v(q,t) = velocity field describing the motion of these lines
- a(q,t) = acceleration field describing the motion of these lines

$$\boxed{-\frac{\partial U}{\partial q}(q,t) = ma(q,t)}$$

"F = ma"

Construction of $U(q,t)$ [given $H(q,p,t)$, E_A]





$H_0 = p^2/2m + V(q,t)$

- vertical lines at $q_1(t)$, $q_2(t)$ … $q_{N-1}(t)$ divide the adiabatic energy shell into N strips of equal phase space volume ($N \to \infty$)
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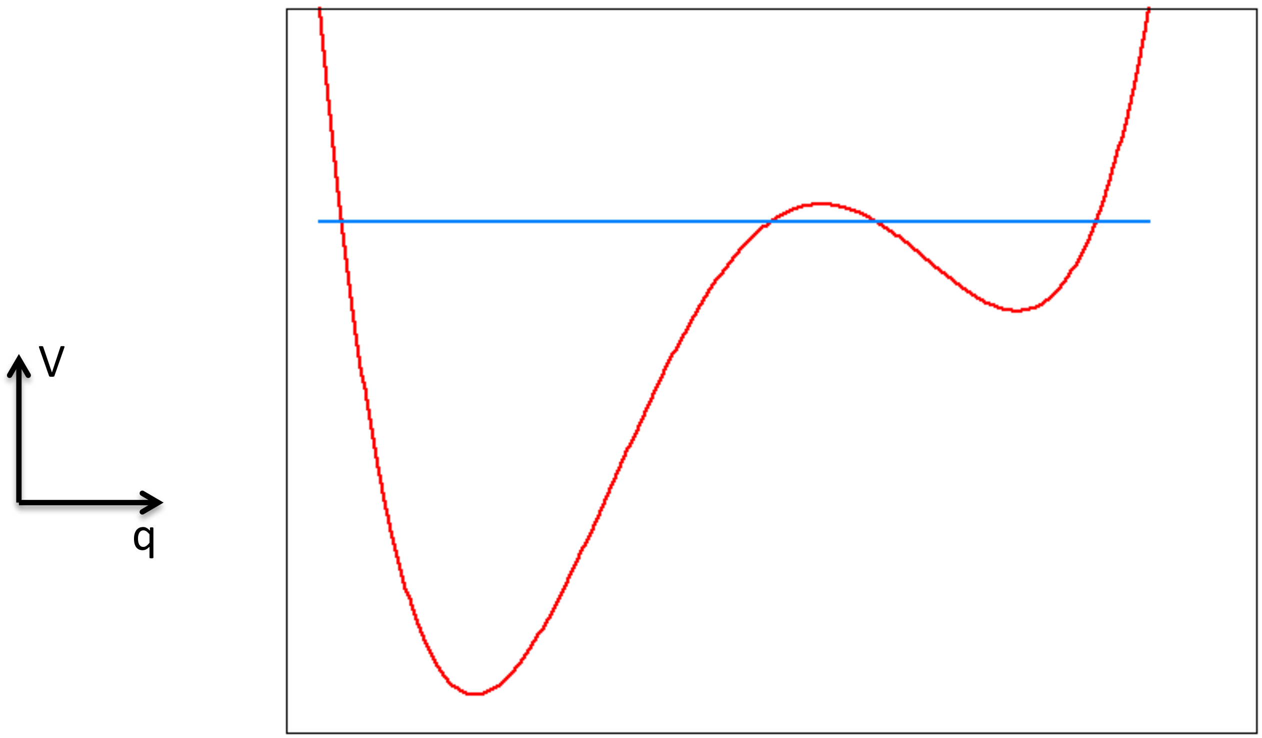
Construction of U(q,t) [given H(q,p,t) , E_A]



$H_0=p^2/2m + V(q,t)$

U(q,t)

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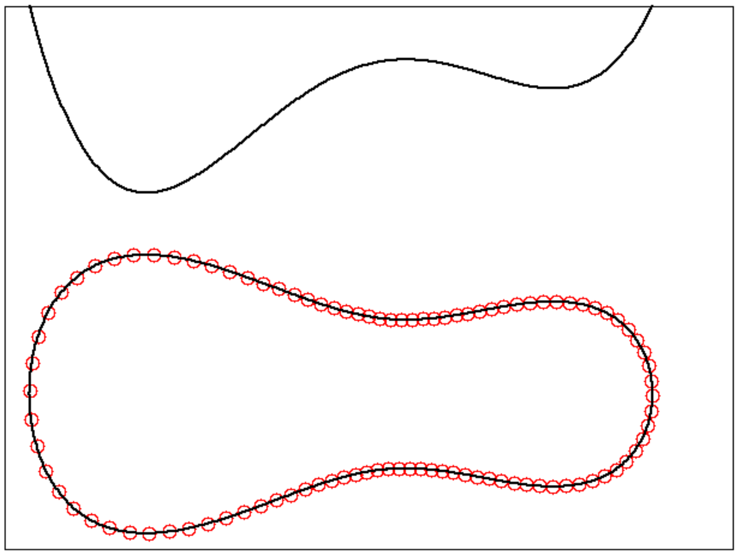
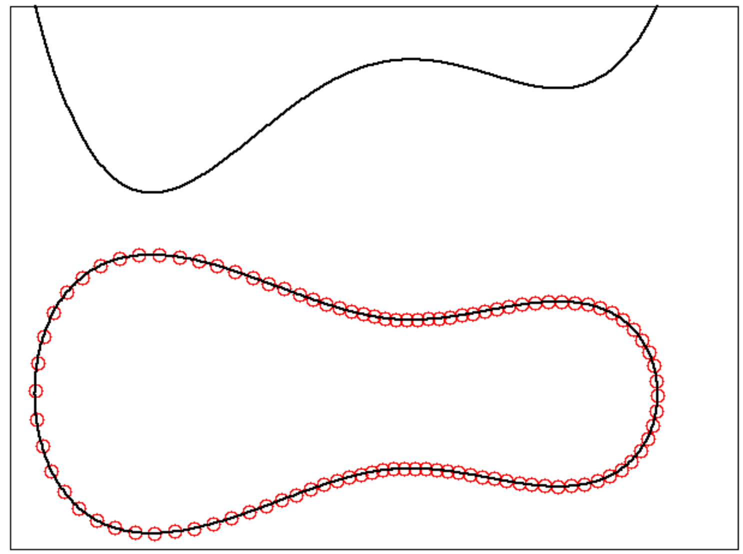
Example: double well

$\tau = 1.0$ (fast)

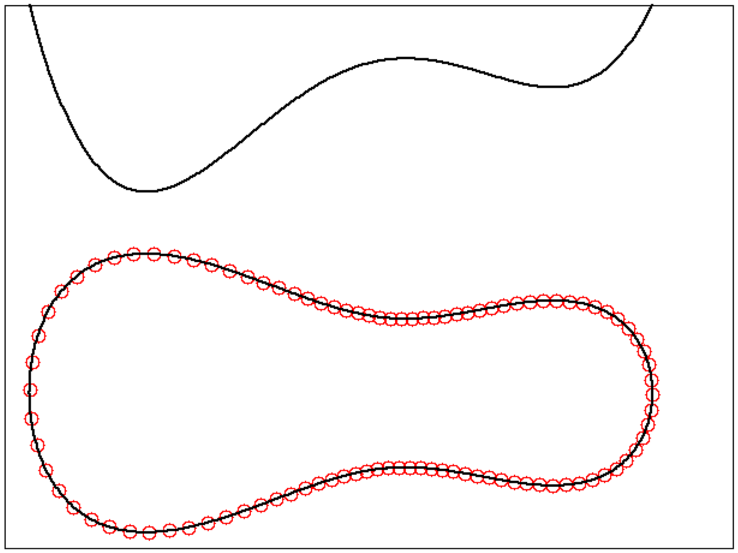




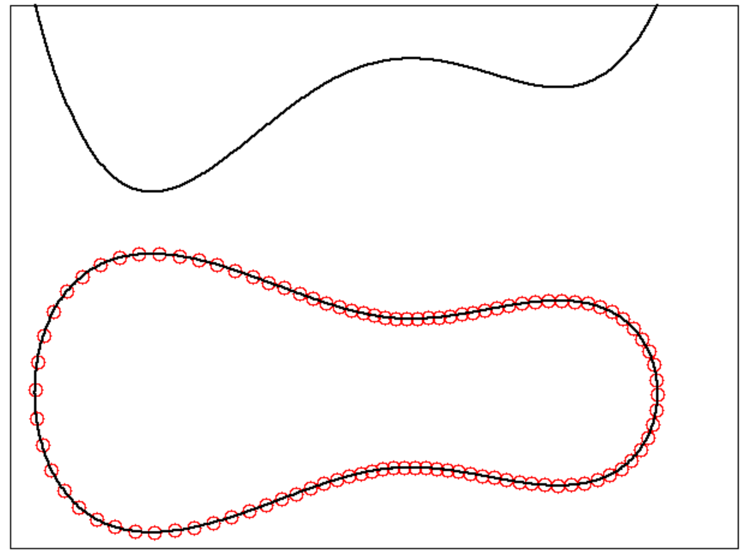
H(q,p,t) H(q,p,t) + U(q,t)

Example: double well

$\tau = 0.1$ (faster)

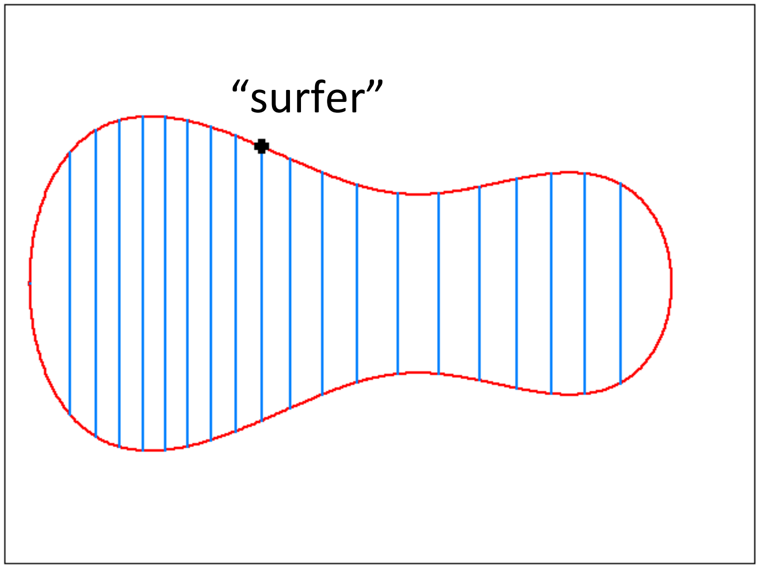


H(q,p,t)



H(q,p,t) + U(q,t)

Derivation



$$H = \frac{p^2}{2m} + V(q,t)$$

$v(q,t)$ = velocity field (flow field) $\quad \dot{q} = v(q,t)$

$a(q,t) = v'v + \dot{v}$ = acceleration field

eqns. of motion for surfer: $\quad \dot{q} = v \;,\; \dot{p} = -pv' \qquad K(q,p,t) = pv(q,t)$

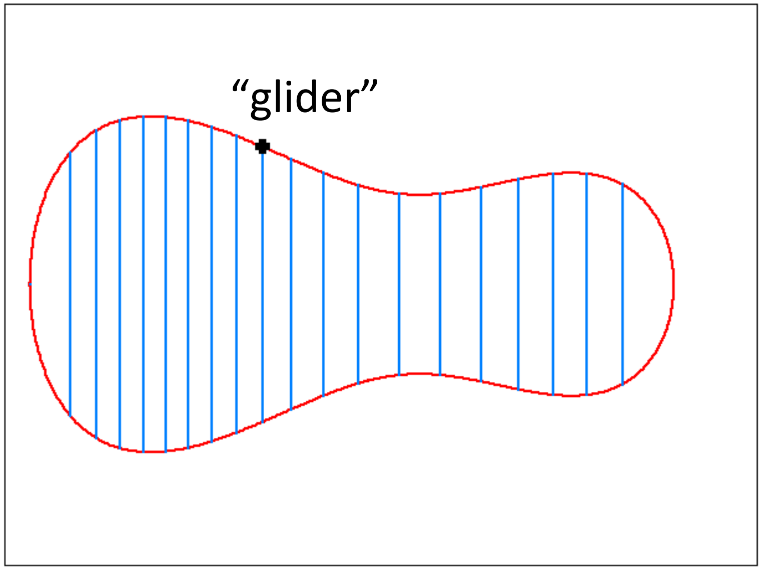
Now define

$$\tilde{H}(q,p,t) = H + K$$

$$= \frac{p^2}{2m} + V(q,t) + pv(q,t)$$

$$\dot{q} = \overbrace{\frac{p}{m}}^{H} + \underbrace{v}_{K} \;,\; \dot{p} = \overbrace{-V'}^{H} \underbrace{- pv'}_{K}$$

Derivation



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Now define $\tilde{H}(q,p,t) = H + K$

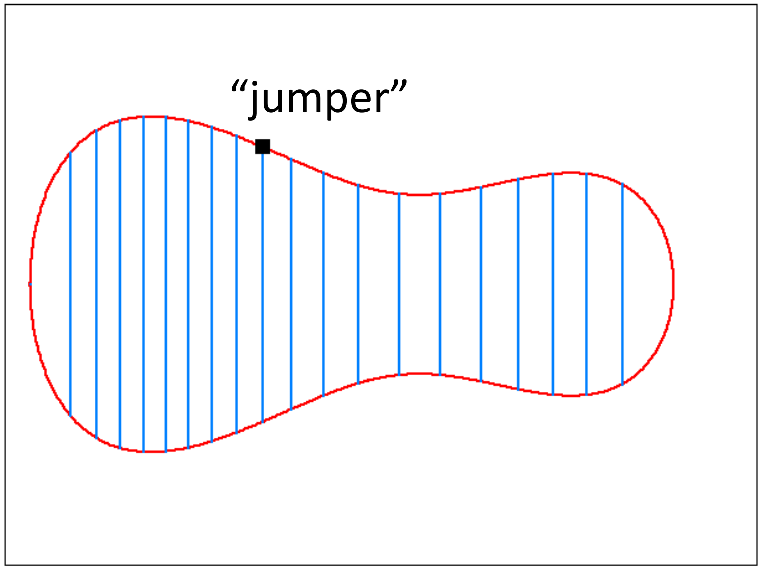
$$= \frac{p^2}{2m} + V(q,t) + pv(q,t)$$

H

$$\dot{q} = \frac{p}{m} + v \;,\; \dot{p} = -V' - pv'$$

K

Derivation



$$H = \frac{p^2}{2m} + V(q,t)$$

$$v(q,t) \quad , \quad a(q,t) = v'v + \dot{v}$$

$$\tilde{H} = \frac{p^2}{2m} + V + pv \qquad \begin{cases} \dot{q} = \dfrac{p}{m} + v \\ \dot{p} = -V' - pv' \end{cases}$$

New coordinates:
$$Q(q,p,t) = q$$
$$P(q,p,t) = p + mv(q,t)$$

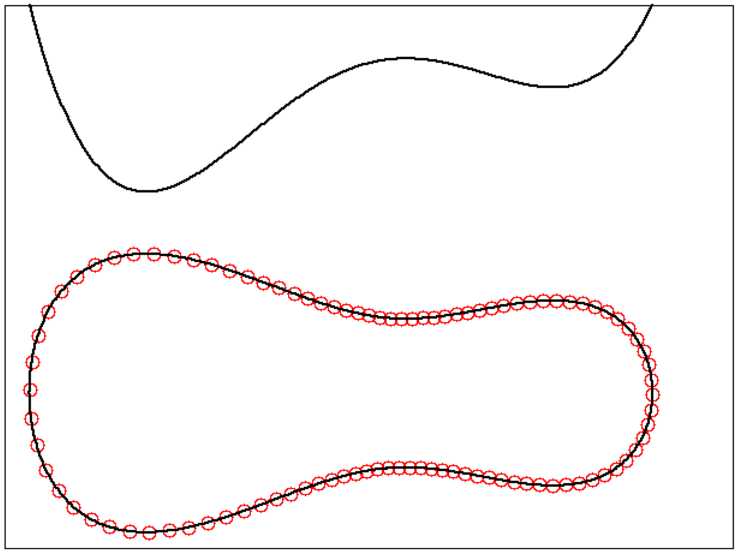
Note: (Q,P) = (q,p) at t=0 and t=τ !

Eqns. of motion: $\dot{Q} = \dfrac{P}{m} \quad , \quad \dot{P} = -V'(Q,t) + ma(Q,t)$

= Hamilton's equations $\bar{H}(Q,P,t) = \dfrac{P^2}{2m} + V(Q,t) + U(Q,t)$

$$\boxed{U' = -ma}$$

Derivation

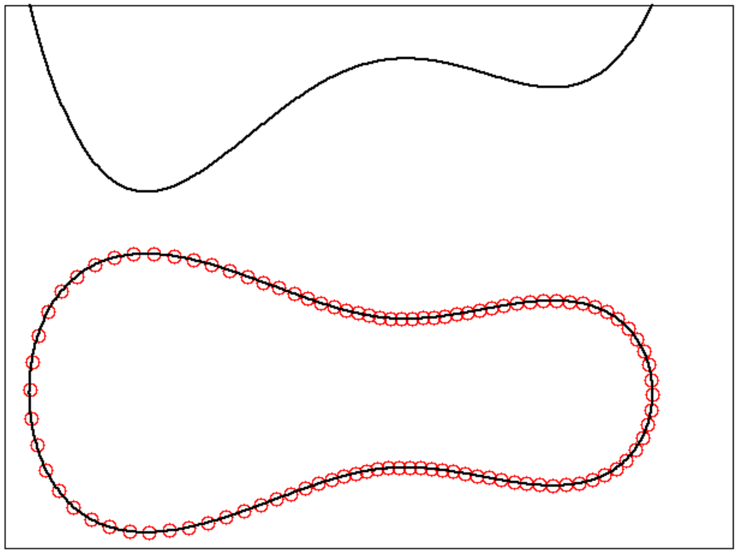


$$H = \frac{p^2}{2m} + V(q,t)$$ original

$$\tilde{H} = \frac{p^2}{2m} + V + pv\left(q,t\right)$$ “non-local”

$$\bar{H} = \frac{p^2}{2m} + V + U\left(q,t\right)$$ “local”

Derivation



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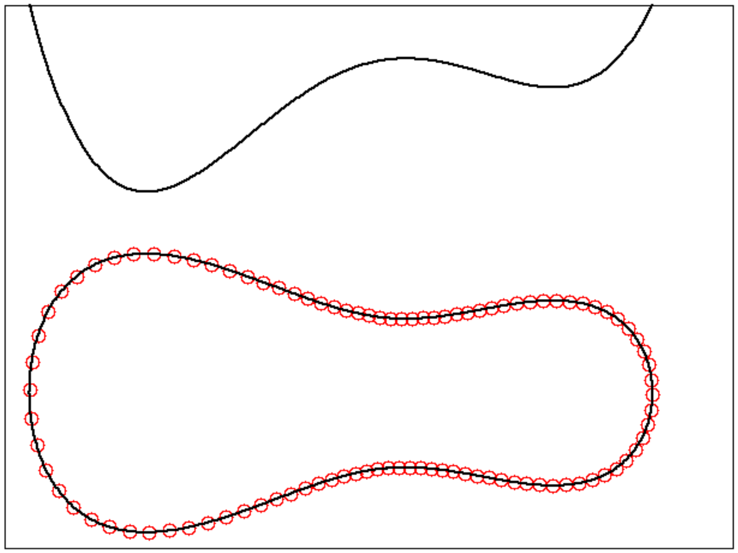

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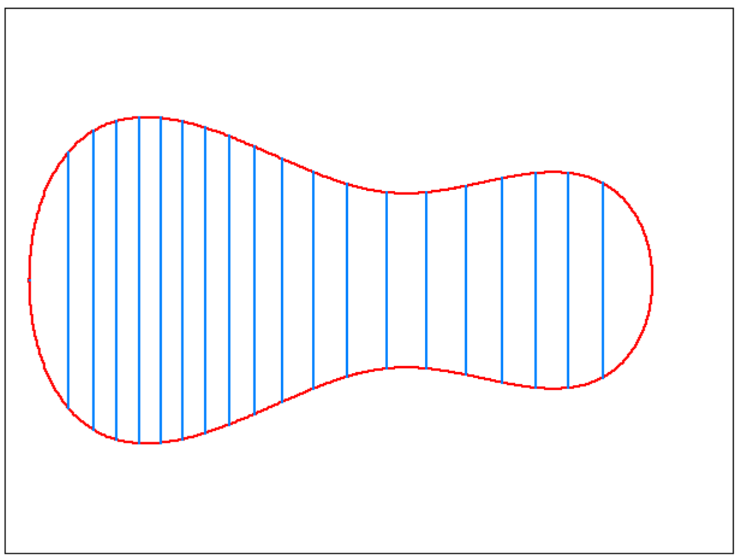


$$v(q,t) \quad , \quad a(q,t) = v'v + \dot{v}$$

$$U' = -ma$$

Special case: *scale-invariant* driving

S. Deffner, C. Jarzynski, A. del Campo
Phys Rev X **4**, 021013 (2014)

$$\boxed{H(q,p,t) = \frac{p^2}{2m} + \frac{1}{\gamma^2} V\left(\frac{q}{\gamma}\right)} \qquad \gamma = \gamma(t) \quad , \quad \dot{\gamma}(0) = \dot{\gamma}(\tau) = 0$$

$$\Longrightarrow \qquad v(q,t) = \frac{\dot{\gamma}}{\gamma} q \qquad a(q,t) = \frac{\ddot{\gamma}}{\gamma} q \qquad U(q,t) = -\frac{m}{2}\frac{\ddot{\gamma}}{\gamma} q^2$$

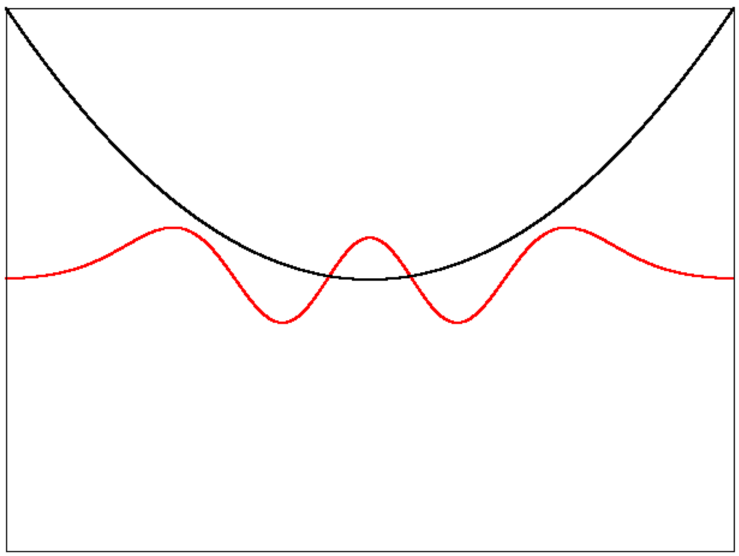
This solution also works for quantum dynamics !

The wavefunction $\quad \psi(q,t) = \phi_n(q,t) \exp\left[\frac{i}{\hbar}\left(\frac{m}{2}\frac{\dot{\gamma}}{\gamma} q^2 - \int_0^t dt' E_n(t')\right)\right]$

is an *exact* solution of the time-dependent Schrödinger equation, under the Hamiltonian $\hat{H}(t) + \hat{U}(t)$, for arbitrary γ(t).

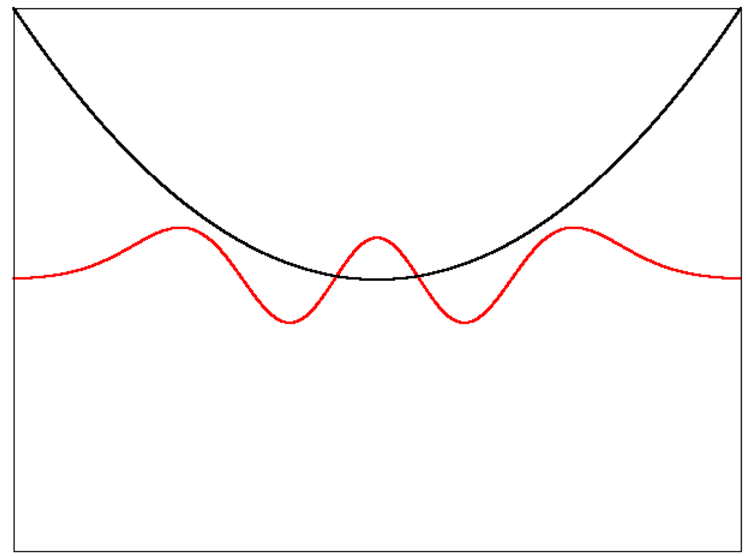
Special case: *scale-invariant* driving

$$\frac{p^2}{2m}+\frac{m\omega^2}{2\gamma^2}\left(\frac{q}{\gamma}\right)^2$$



without counter-diabatic term

$$\frac{p^2}{2m}+\frac{m\omega^2}{2\gamma^2}\left(\frac{q}{\gamma}\right)^2-\frac{m}{2}\frac{\ddot{\gamma}}{\gamma}q^2$$

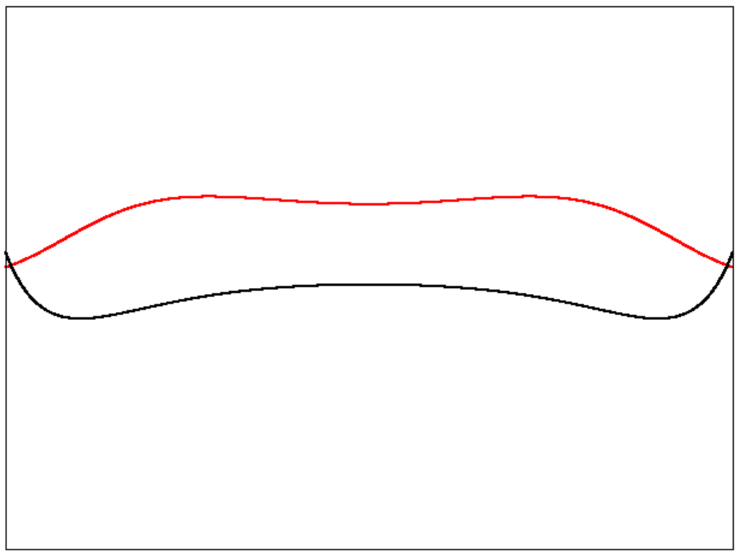


with counter-diabatic term

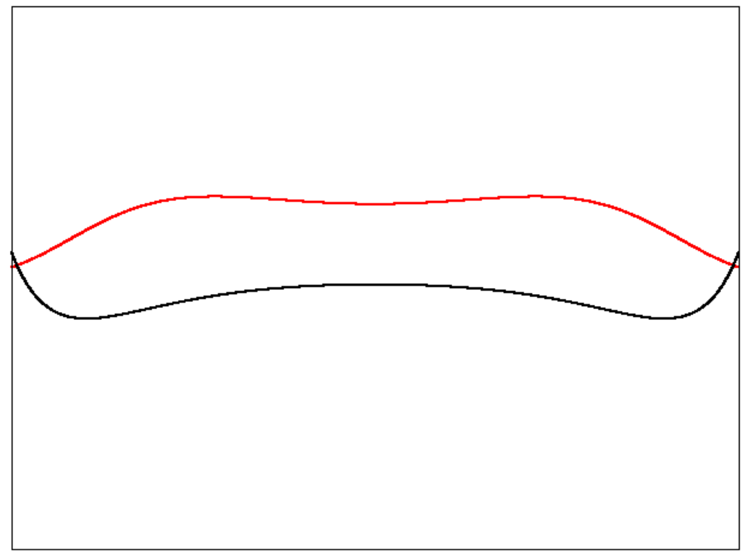
X. Chen *et al*, *Phys Rev Lett* **104**, 063002 (2010)
S. Deffner, C.J., A. del Campo, *Phys Rev X* **4**, 021013 (2014)

What about non-scale-invariant driving?

S. Masuda & K. Nakamura
Phys Rev A **78**, 062108 (2008)
ground state only !



without counter-diabatic term



with counter-diabatic term

system begins & ends in ground state