Dynamics of active nematics under confinement

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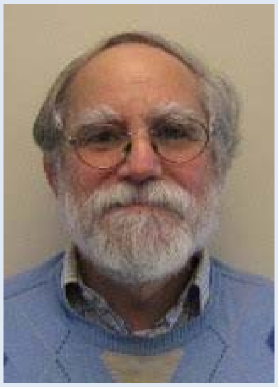
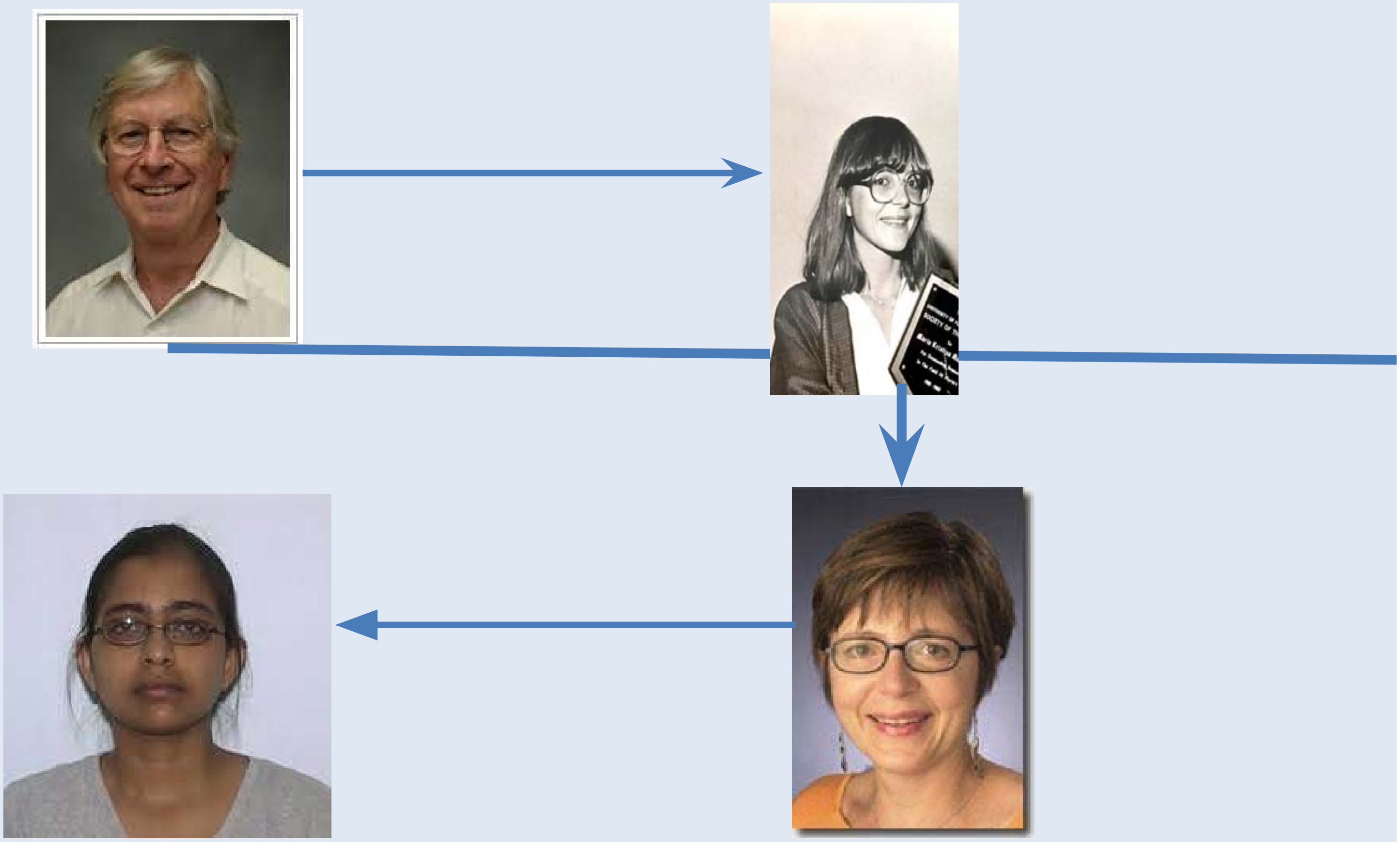
Introduction and Context



Introduction and Context



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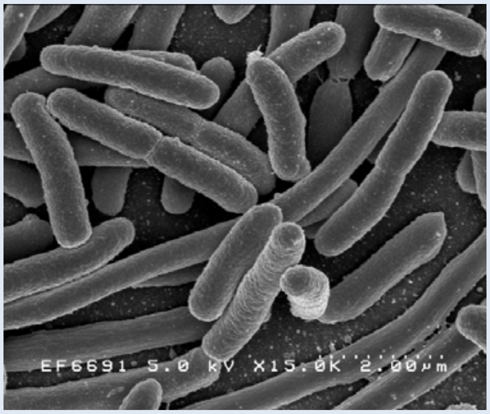
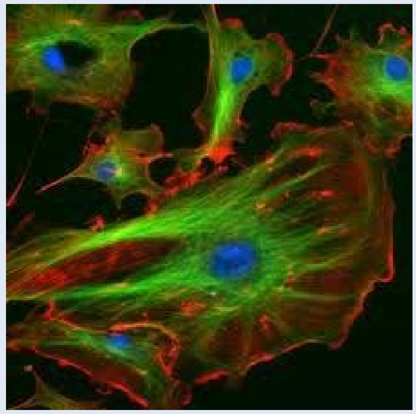
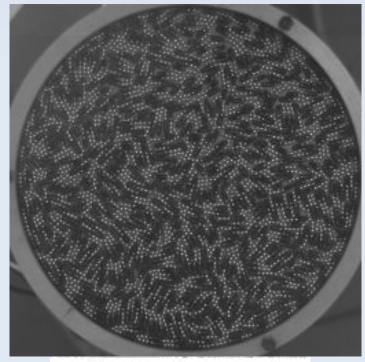
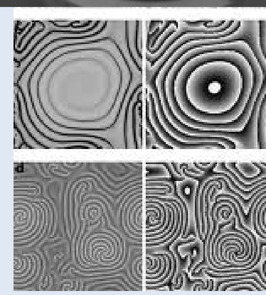


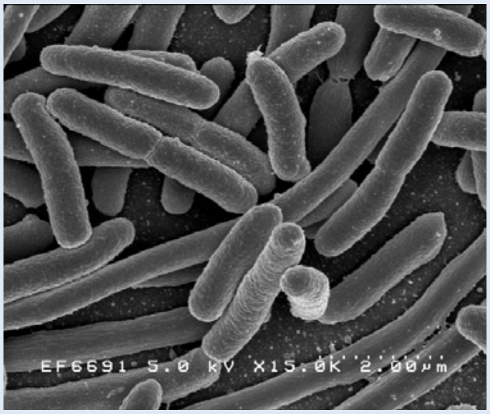


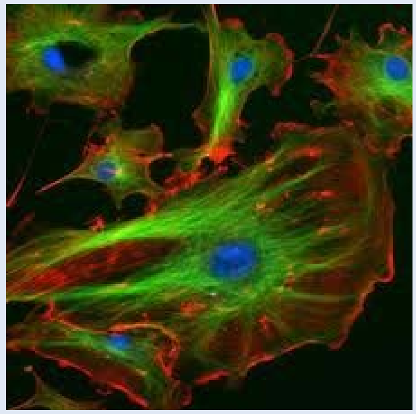


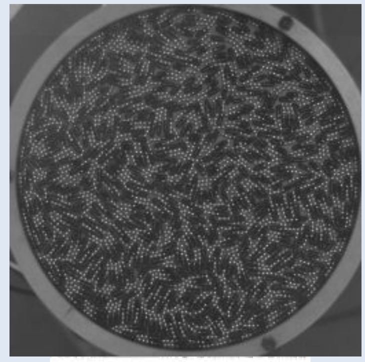


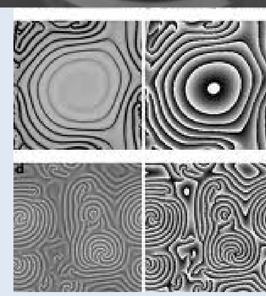
Introduction and Context

Active Materials









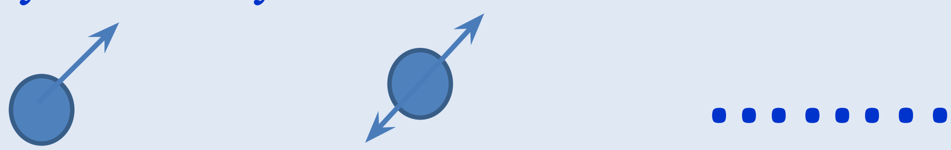
- A soft material
- Each “molecule” is an engine locally coupled to others
- Collectively do work at the macroscale

Introduction and Context

- A theoretical approach that is “don’t care” on the particular microscopics – think Navier-Stokes

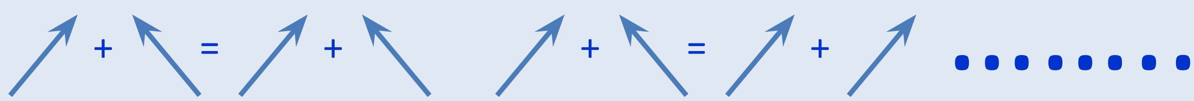
- Classify fluids based on symmetry

- Symmetry of activity



Polar　　　Dipolar/Nematic

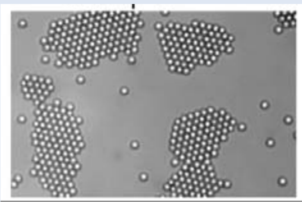
- Symmetry of interactions



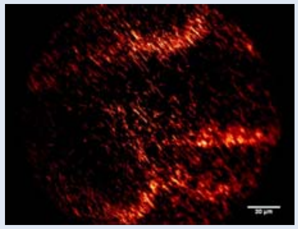
Don’t care/isotropic　　　Polar

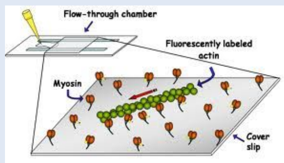
Introduction and Context

Examples

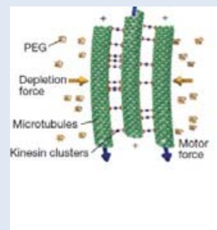


Polar drive
/isotropic
interactions





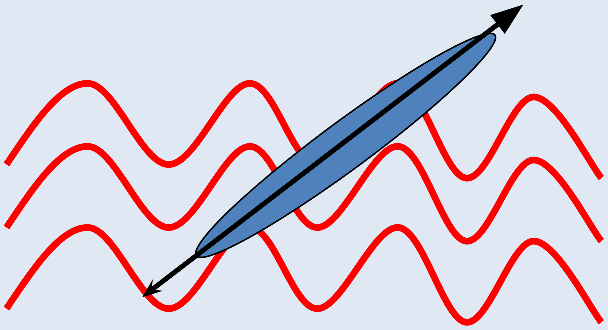
Polar drive
/Polar
interactions

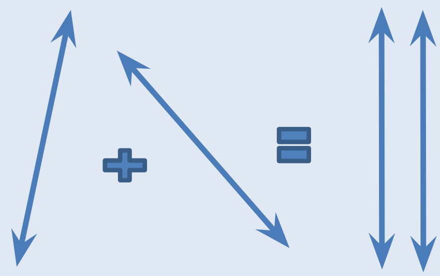


(Movie) Experimental movie

Nematic drive
/Nematic
interactions

Part 1 : Hydrodynamics of active nematics





Hydrodynamic Description

Density of Particles

$$\rho\left(\mathbf{r},t\right)$$

Nematic Order Parameter

$$Q_{\alpha\beta}\left(\mathbf{r},t\right)=\left\langle \hat{u}_\alpha\hat{u}_\beta-\frac{1}{2}\delta_{\alpha\beta}\right\rangle$$

Fluid velocity

$$\mathbf{u}\left(\mathbf{r},t\right)$$

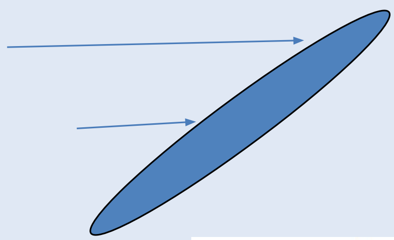
Part 1 : Hydrodynamics of active nematics

- Start with an equilibrium nematic fluid

$$\partial_t Q_{\alpha\beta} = -\frac{\delta F}{\delta Q_{\alpha\beta}}$$

- Gradient descent dynamics on some free energy landscape

- Under some imposed inhomogeneous flow



$$\partial_t \mathbf{r} = \mathbf{u}\left(\mathbf{r}, t\right)$$

$$\partial_t \hat{u} = \overleftrightarrow{\Omega} \cdot \hat{u} + \frac{h^2 - 1}{h^2 + 1}\left(I - \hat{u}\hat{u}\right) \cdot \overleftrightarrow{E} \cdot \hat{u}$$

$$\Omega_{\alpha\beta} = \frac{1}{2}\left(\right.$$

$$Q_{\alpha\beta}\left(\mathbf{r}, t\right) = \left\langle \hat{u}_\alpha \hat{u}_\beta - \frac{1}{2}\delta_{\alpha\beta} \right\rangle$$

$$E_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha\right)$$

Part 1 : Hydrodynamics of active nematics

- Start with an equilibrium nematic fluid

$$\partial_t Q_{\alpha\beta} = -\frac{\delta F}{\delta Q_{\alpha\beta}}$$

- Gradient descent dynamics on some free energy landscape

- Under some imposed inhomogeneous flow

$$\partial_t Q_{\alpha\beta} + \mathbf{u} \cdot \nabla Q_{\alpha\beta} + \Omega_{\alpha\gamma} Q_{\gamma\beta} - Q_{\alpha\gamma}\Omega_{\gamma\beta} - \lambda E_{\alpha\gamma} = -\frac{\delta F}{\delta Q_{\alpha\beta}}$$

- Now to confront the equation for the flow

Part 1 : Hydrodynamics of active nematics

- Start with Navier-Stokes

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \eta\nabla^2\mathbf{u} + \nabla\cdot\overleftrightarrow{\sigma}^r$$

$$\nabla\cdot\mathbf{u} = 0$$

- Add reactive stresses : chosen to make theory dissipative

- This theory is ksson or Edward Berry theory f

$$E = \int\left\{\frac{1}{2}u^2 + F\left[Q\right]\right\}$$

- $$\frac{dE}{dt} = \int\left\{\begin{array}{c} u\cdot\left(-\mathbf{u}\cdot\nabla\mathbf{u} - \nabla p + \eta\nabla^2\mathbf{u} + \nabla\cdot\overleftrightarrow{\sigma}^r\right) \\ +\left(-\mathbf{u}\cdot\nabla Q - \Omega Q + Q\Omega + \lambda E - \frac{\delta F}{\delta Q}\right) : \frac{\delta F[Q]}{\delta Q}\end{array}\right\}$$

$$\frac{dE}{dt} = -\int\left\{\eta\nabla\mathbf{u} : \nabla\mathbf{u} + \frac{\delta F}{\delta Q} : \frac{\delta F}{\delta Q}\right\}$$

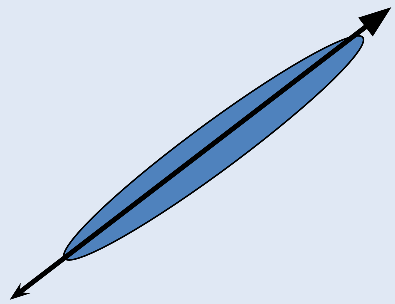
Part 1 : Hydrodynamics of active nematics

- Start with Nematohydrodynamics

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \eta \nabla^2 \mathbf{u} + \nabla \cdot \overleftrightarrow{\sigma}^r$$

$$\nabla \cdot \mathbf{u} = 0$$

- Particles are exerting forces



$$\sigma^a = \alpha Q$$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \eta \nabla^2 \mathbf{u} + \nabla \cdot \overleftrightarrow{\sigma}^r + \alpha \nabla \cdot \overleftrightarrow{Q}$$

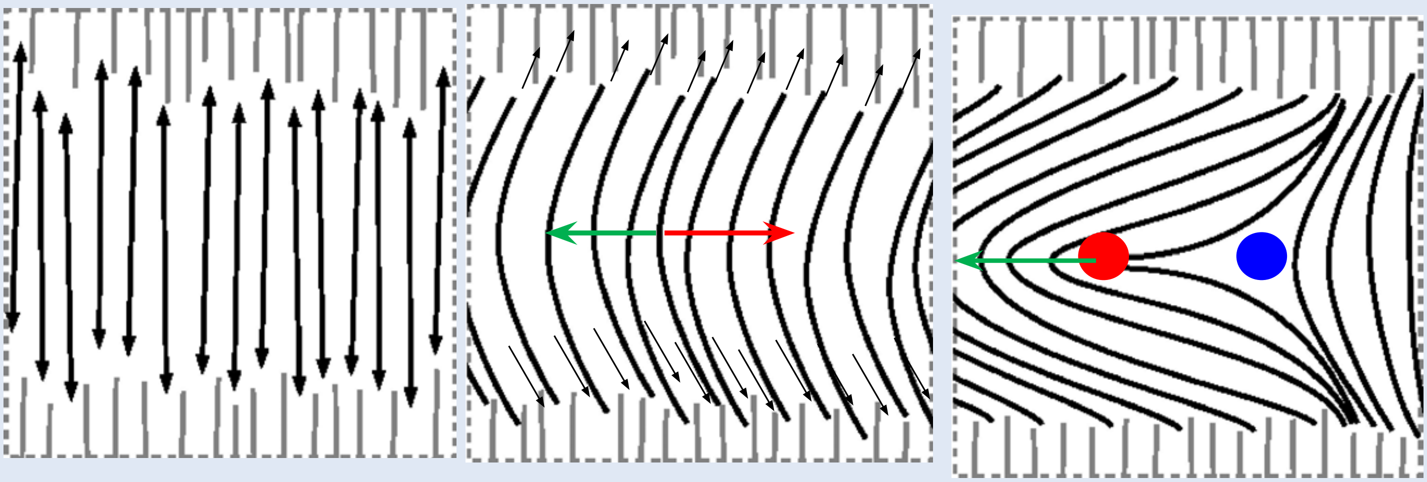
Part 1 : Hydrodynamics of active nematics

$$\partial_t Q_{\alpha\beta} + \mathbf{u}\cdot\nabla Q_{\alpha\beta} + \Omega_{\alpha\gamma}Q_{\gamma\beta} - Q_{\alpha\gamma}\Omega_{\gamma\beta} - \lambda E_{\alpha\gamma} = -\frac{\delta F}{\delta Q_{\alpha\beta}}$$

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \eta\nabla^2\mathbf{u} + \nabla\cdot\overleftrightarrow{\sigma}^r + \alpha\nabla\cdot\overleftrightarrow{Q}$$

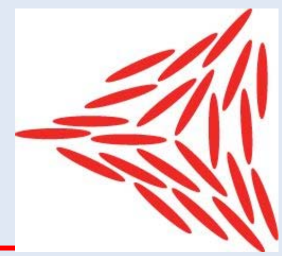
$$\nabla\cdot\mathbf{u} = 0$$

Part 2 : Phenomenology of the theory



+½ defect

-½ defect



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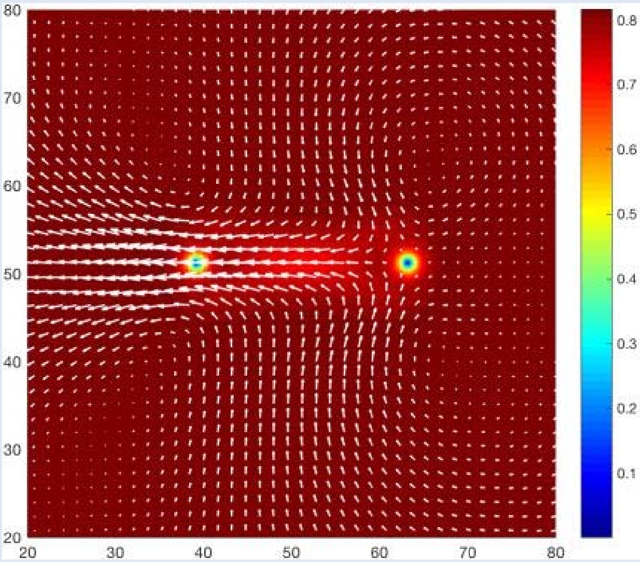
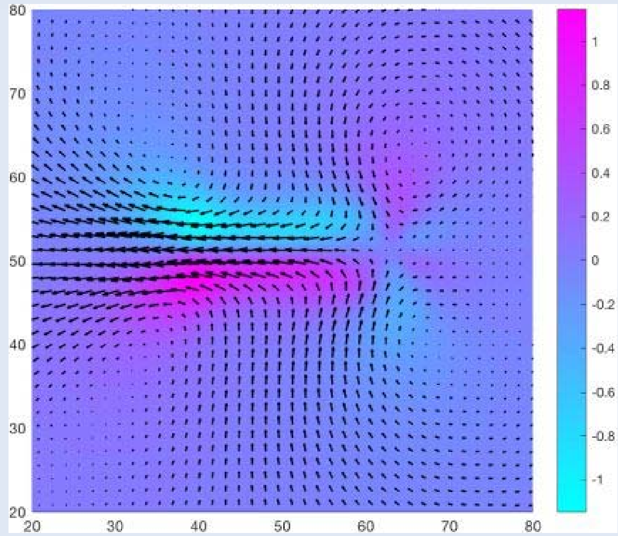


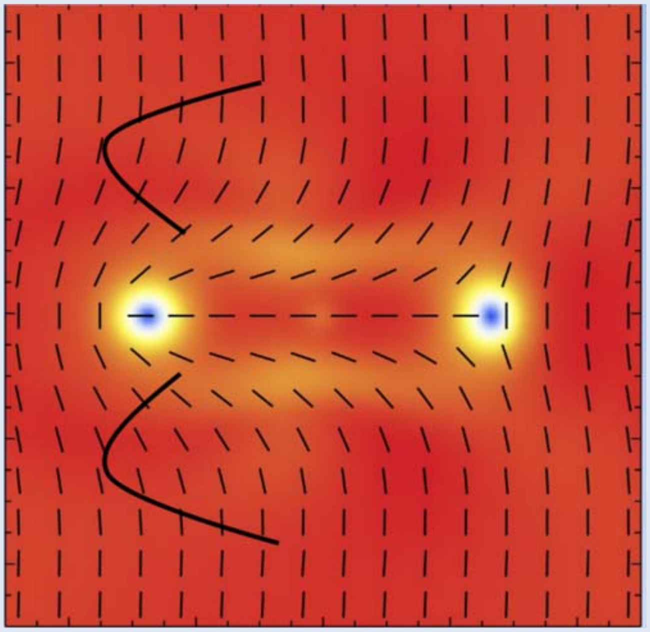
Degree of order

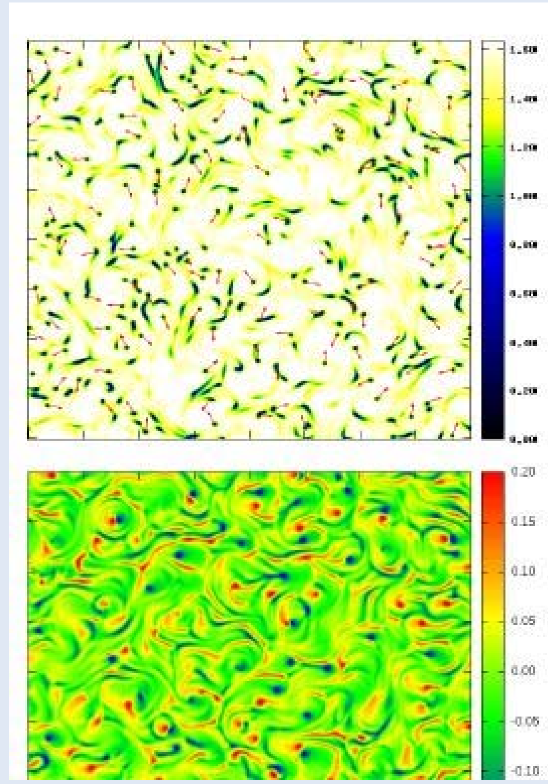


vorticity

Part 2 : Phenomenology of the theory

- Basic phenomenology of the theory



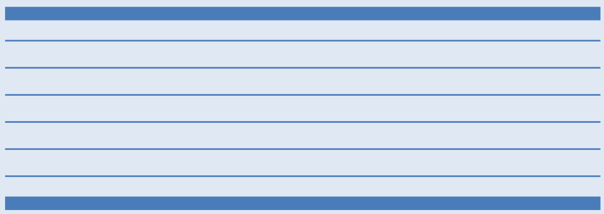


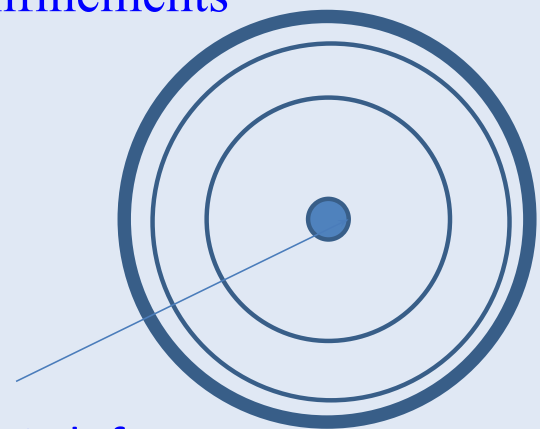
[Experimental movie]

Part 3 : Confined Active nematics

- Want to put these things in a box

- Two cases : channels and circular confinements

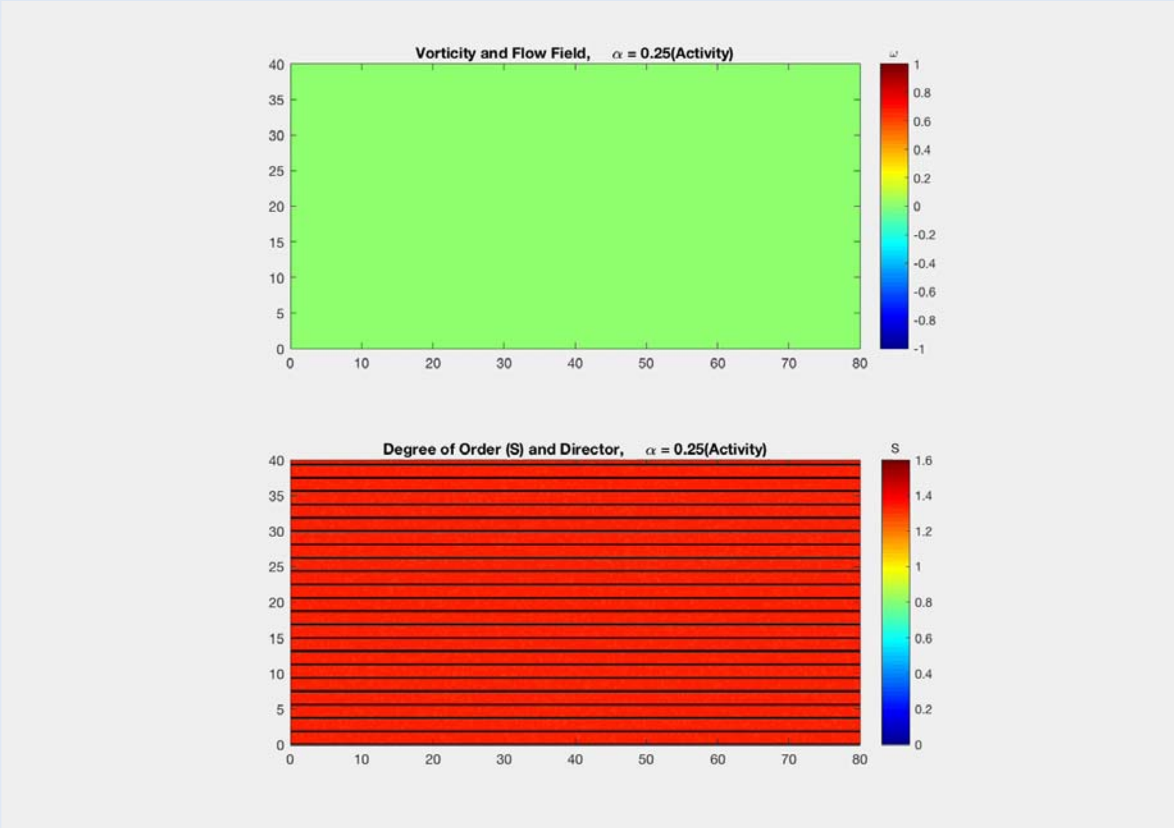




Topologically requires +1 defect

- The walls are no slip for the fluid – they tame vorticity

Part 3 : Confined Active nematics



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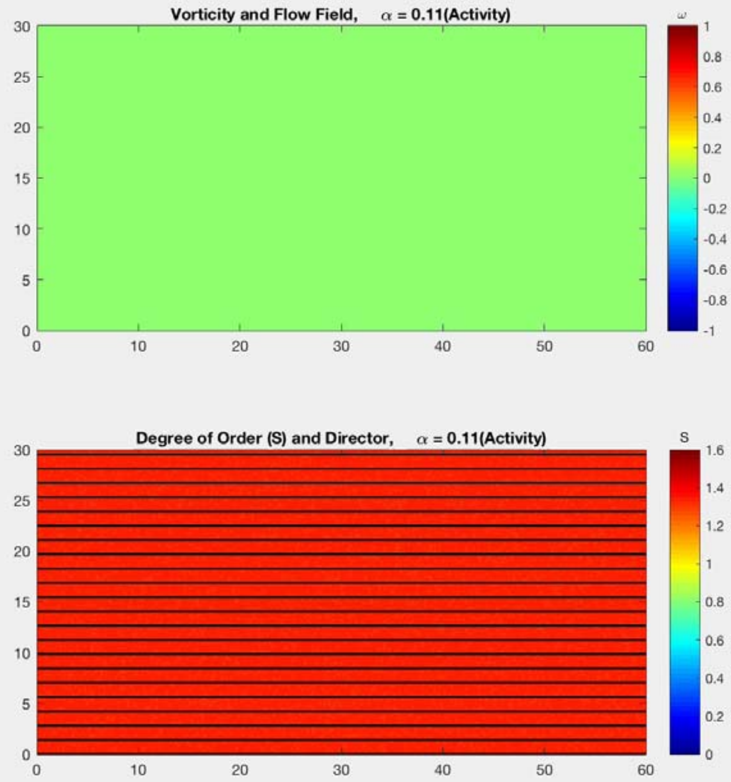

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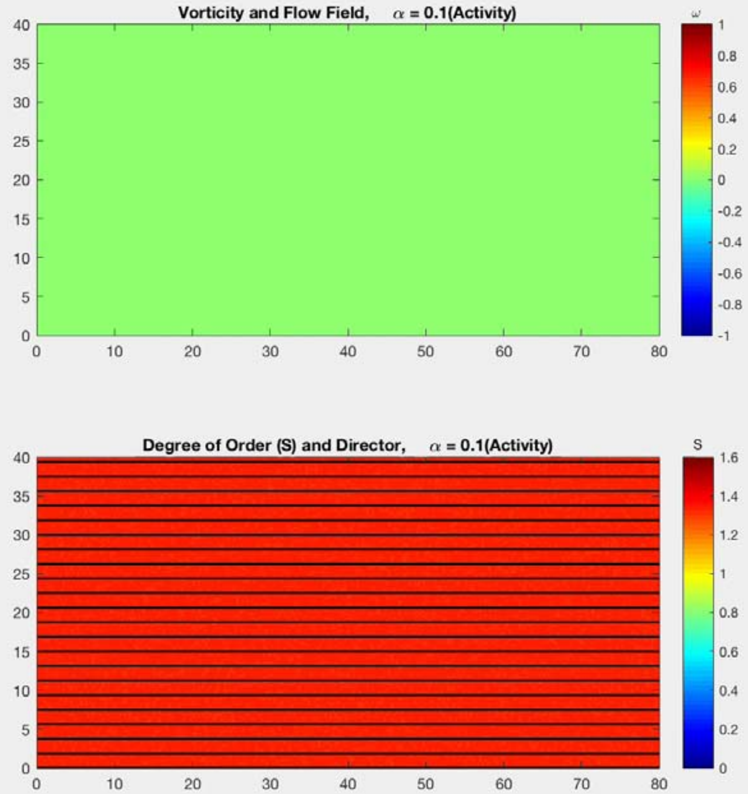

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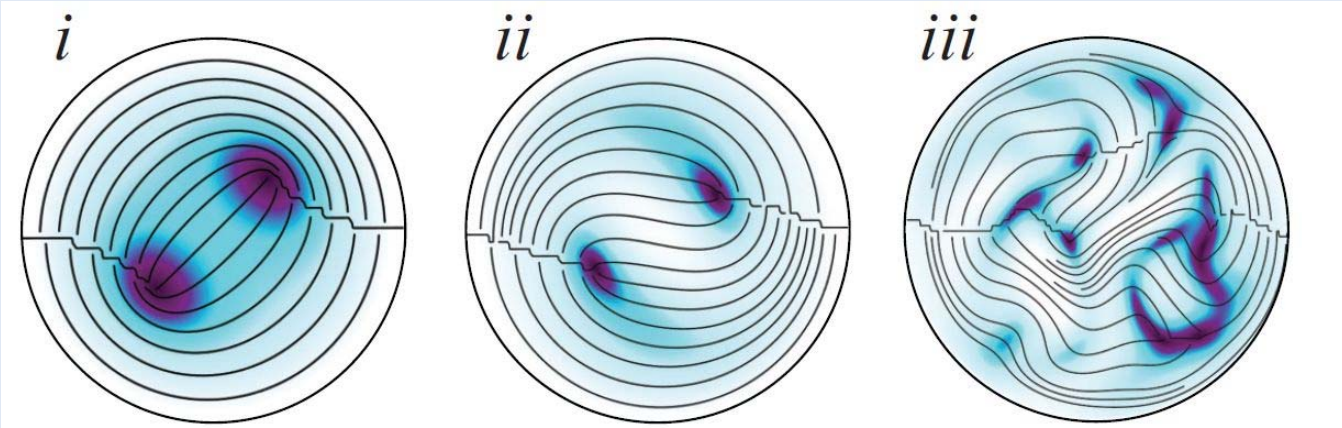
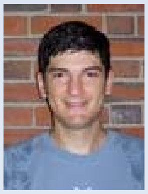


Static Dipole Yin-Yang Turbulent

Former Students



Gabe Redner

Elias Putzig

Minu Varghese

Zack Sustiel

Grad student

Undergraduate student

Post Docs

Abhijeet Joshi

Mike Norton

Arvind Baskaran

Mike Hagan

Seth Fraden

Zvonimir Dogic