# Determinism, Indeterminism and the Statistical Postulate

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# Background

- What am I doing here?
- I was trained in Physics
  - Laurea in Milan
  - Doctorate in Genova with Nino Zanghì
    - Worked with Detlef Dürr and Shelly Goldtsein
- I switched to Philosophy
  - Doctorate at Rutgers with Tim Maudlin
    - Continued to work with Shelly, Detlef, Nino, and Rodi Tumulka

# Background

- Context: Boltzmann's account of thermodynamics as described by David Albert's book "Time and Chance" (2001)
- Main ingredients:
  - The dynamics
  - The past hypothesis (PH)
  - The statistical postulate (SP)
- Why is the statistical postulate needed?
  - To ground probabilities and thus make Boltzmann's account explanatory

# Albert's argument for GRW

- Assume that, within the quantum domain, we have a similar structure.
- Then, if the fundamental theory of the world is indeterministic, and in particular like the GRW theory, then one can eliminate the statistical postulate:
  - Because now statistical probabilities are quantum mechanical probabilities
- If so, then GRW should be preferred to deterministic theories on the basis of parsimony considerations
  - Two posits (dynamics, past hypothesis) instead of three (dynamics, past hypothesis **and** statistical postulate)

# The main point of this talk: one may <u>never</u> need a statistical postulate

- Using Shelly's teachings, one could that:
- 1) Assuming the SP, there is no need of a probability measure to make Boltzmann's account explanatory. Instead, one can use a <u>typicality measure</u> instead.
- 2) The typicality measure is <u>not postulated</u>: it comes from the dynamics using symmetry constraints (stationarity, generality).
- 3) Deterministic and indeterministic theories share the same <u>unifying approach</u> to statistical explanation.
  - If so, there is no reason, in this context, to think that GRW provides a better explanatory framework

# Review of Albert's reading of Boltzmann - 1

- Purpose: derive thermodynamic laws in terms of microscopic Newtonian dynamics
- Problems:
  - Technical problems:
    - Forces between the particles unknown;
    - Too many particles for exact computation
      - Solution: statistical methods:
        - If $N$ is large, one can use mathematical techniques to derive information about macroscopic systems even without a solution to the above problems
  - Macroscopic processes are irreversible, microscopic ones are reversible
  - → Boltzmann!

# Review of Albert's reading of Boltzmann - 2

- Microscopic view:
  - Microstate $X = (r_1, \ldots . r_N, v_1, \ldots . v_N)$ in phase space
  - Partition of phase space into macrostates
  - Macrostate $\mathrm{M}(X)$:
    - Set of microstates that are macroscopically indistinguishable
      - → Macroscopic view
      - Incomplete:
        - Many $X$ for one $M$ → given $M$, we do not know which $X$
    - Macroscopic properties (e.g. temperature): slowly varying on the microscale

# Review of Albert's reading of Boltzmann - 3

- Equilibrium state: the **biggest** macrostate of all
  - There are many more ways to be distributed as to have, say, uniform temperature than not
- Being so big explains:
  - 1) Why macroscopic systems not in equilibrium <u>evolve towards</u> equilibrium
    - Because at any time there are many more ways to be in equilibrium than not, the <u>vast majority</u> of microstates not in equilibrium will assume an equilibrium distribution, eventually
  - 2) Why macroscopic systems which have reached equilibrium, <u>do not leave it</u>
    - Because at any time there are many more ways to be in equilibrium than not, the <u>vast majority</u> of microstates in equilibrium will continue to assume one of the many possible equilibrium distributions
- Possibility of 'abnormal' (exceptional/anti-thermodynamical) microstates that do not do 1 or 2! → they are few

# Review of Albert's reading of Boltzmann - 4

- Boltzmann: define Entropy $\propto$ size in phase space
- Thus:
  - Entropy increases because microstates will go from smaller macrostates to larger ones
    - Or at least the vast majority of them
- In this way, using the dynamics alone, Boltzmann has shown that
  - Entropy increases toward the future for most microstates

# Review of Albert's reading of Boltzmann - 5

- The need for a Past Hypothesis (PH):
  - Since the microscopic dynamics is time reversal invariant, entropy will tend to increase in the future, but **also in the past**
  - And this is not empirically adequate:
    - we need to eliminate the second possibility
  - One of the option:
    - The PH: postulate an asymmetric initial condition such as a universe with an extremely low initial entropy

# Review of Albert's reading of Boltzmann - 6

- The need for a Statistical Postulate:
  - With the dynamics and the PH, we arrived at:
    - 1-Entropy increases toward the future <u>for most microstates</u>
  - However, in order to explain, one should say instead:
    - 2-Entropy has <u>a high probability</u> of increasing toward the future
  - In order to go from 1 to 2, one needs that all microstates count <u>equally</u>:
    - The uniform measure
  - The usual <u>justifications</u> for the uniform measure are in terms of ignorance:
    - They appeal to the <u>principle of indifference</u>
  - But this makes <u>no sense,</u> says Albert:
    - How can our ignorance influence the motion of physical things?
  - Thus we need to <u>postulate</u> the uniform measure

# Review of Albert's extension of B's approach to quantum mechanics

- Main idea: GRW can provide a dynamical explanation of the SP:
  - An 'abnormal' (i.e. entropy-decreasing) microstate will be surrounded by normal states (since they are the majority) and hence it is highly unstable:
    - Perturb it a little, and it will go into a normal state
  - The perturbation provided by the wave-function collapse in GRW will make abnormal states 'jump' into normal ones
  - GRW theory randomly generates sets of possible states for the wave-function, each of them with a quantum probability measure
  - Thus, no SP is required.
- If so, a GRW-based statistical explanation only have two postulates, rather than three and as such should be preferred.

# Reflections on the Statistical Postulate: Towards Typicality

- Two remarks on Albert's take on the SP:
  - 1) Statistical explanation needs to have <u>probabilities</u> in order to be explanatory
  - 2) The SP has to be <u>postulated</u> in addition to the dynamics
- In my opinion (following Shelly & co.): none of them is necessarily the case
- As a response to 1: Typicality
  - Typicality translates
    - 1-Entropy increases toward the future for <u>most microstates</u>
      - as
    - 2′-Entropy <u>typically</u> increases toward the future
  - A typicality measure defines the meaning of ‘ vast majority’ as ‘the set of exception to the entropy-increasing behavior, as measured by the typicality measure, is small’

# Reflections on the Statistical Postulate – Typicality - 1

- Typicality is <u>not probability</u>:
  - All that is needed is that the exceptions as measured by $\mu$ are small
    - No need to satisfy probability axioms, or additivity or the like
- Typicality is <u>explanatory</u>:
  - Why is equilibrium typical? Because $\frac{\mu(M_{non\ eq})}{\mu(M_{eq})} \ll 1$
  - 1-Why systems in equilibrium stay in equilibrium? Because $\frac{\mu(E)}{\mu(M_{eq})} \ll 1$
  - 2-Why systems outside equilibrium will go there? Because $\frac{\mu(E)}{\mu(\Gamma)} \ll 1$
  - Why does entropy (almost) never decrease?
    - Because for states in equilibrium, constant-entropy behavior is typical (see #1)
    - Because for states outside of equilibrium, entropy-increasing behavior is typical (see # 2).

# Reflections on the Statistical Postulate – Typicality - 2

- Wait! Does that mean that one can say that it is typical for entropy to increase but not that it is probable?!? … No
- If so, how can typicality ground **probability talk**?
  - Degrees of belief:
    - Typicality can be interpreted as the degree of belief of an observer making inferences on the probability of entropy increasing.
    - Nonetheless, the appeal to one's beliefs is not an explanation for the reason why a natural phenomenon (namely entropy increasing) occurs.
  - Empirical Frequencies:
    - If we perform repeated experiments we obtain an empirical distribution, which present statistical patterns.
    - Typicality is such to show that the theoretically predicted distributions and the empirical ones are typically close.
      - In this way explaining the observed distributions

# Reflections on the Statistical Postulate – Typicality - 3

- Explanation based on typicality:
  - **To understand why there is a given regularity is to understand why the regularity is typical**
    - True for the vast majority of initial states (but not for all)
- No further explanation is required if some phenomenon is typical:
  - Explanations are needed for <u>deviant behaviors</u> instead
- An explanation which is true for most initial conditions is <u>acceptable:</u>
  - Unacceptable ones: using *special initial conditions* one can explain everything!
- <u>Predictive power</u> of typicality:
  - If a property is typical, then one should <u>expect</u> (i.e. predict) to have this property in other similar systems as well

# Reflections on the Statistical Postulate – Origin of the Typicality Measure - 1

- Shelly & co: the typicality measure is defined by the dynamics, so no SP is needed (my reconstruction) –
  - P1: The purpose of the typicality measure is to count microstates
  - P2: One can do this at different times
  - P3: One should not privilege one time over any other
  - subC: Thus the typicality measure should be time-translation invariant (TT)
  - P4: If the dynamics is Hamiltonian, the typicality measure should be independent of the specific form of the Hamiltonian (generality)
  - P5: The only measure satisfying these two requirements is the uniform measure
  - C: Thus the uniform measure is the typicality measure (for Hamiltonian systems).

# Reflections on the Statistical Postulate – Origin of the Typicality Measure – 2

- Objections (Bricmont):
  - 1) There are counterexamples:
    - In given chaotic systems the typicality measure is not the Liouville measure, which is not even stationary, but the Sinai-Ruelle-Bowen (SRB) measure
  - Possible reply: this does not contradict the argument
    - It makes sense that the typicality measure for chaotic systems is not the Liouville measure:
      - in these systems the points in space should not be counted equally: only the ones on the attractor influence the behavior of the system, and thus one should use the typicality measure only on them.
    - Thus, the relevant measure is only on the attractor.
    - The stationary measure on the attractor is the SRB measure
    - So, the typicality measure is the SRB measure

# Reflections on the Statistical Postulate – Origin of the Typicality Measure - 3

- Objections (Bricmont):
  - 2) TT is not  the 'right' notion:
    - A theory of scientific explanation should also capture our intuitions
    - The notion of TT is never part of our intuitions while explaining
    - Thus, TT cannot genuinely be part of a scientific explanation
  - Possible reply:
    - TT may be counterintuitive
    - However, it is connected with the idea that no temporal instant should be privileged
    - *This* is intuitive
    - Typicality uses this intuition, through its implementation with the sophisticated notion of TT to explain.
      - Einstein (1936) : “the whole of science is nothing more than a refinement of our everyday thinking”.
      - This refinement can lead us far from intuition <u>without losing </u>its legitimacy.

# Reflections on the Statistical Postulate – Origin of the Typicality Measure - 4

- To me, if there is a weakness of the previous argument, it is the requirement of generality (or genericity) of the measure:
  - P4: If the dynamics is Hamiltonian, the typicality measure should be independent from the specific form of the Hamiltonian
- Potential argument against P4:
  - Why as for it? There is only one true Hamiltonian for the universe, so care about others?
- Possible reply:
  - Subsystems: effective Hamiltonian
    - From true $H$ → a bunch of TT-measures
    - From $H_{eff}$ → a bunch of other TT-measures
    - Since both $H$ and $H_{eff}$ should give the same empirical results, then one should require that the measure they use be the same, hence the Liouville measure.

# Reflections on the Statistical Postulate – summary so far

- We have arrived at the following:
  - Typicality (not necessarily probability) is enough to explain;
  - The typicality measure (namely the ones that proves empirically adequate) may be inferred from the dynamical laws using symmetries considerations (TT, generality), rather than postulated.
- If so, the ingredients now are reduced to the following:
  - The laws of motion (which determine the typicality measure on phase space)
  - The Past Hypothesis.

# Indeterminism - 1

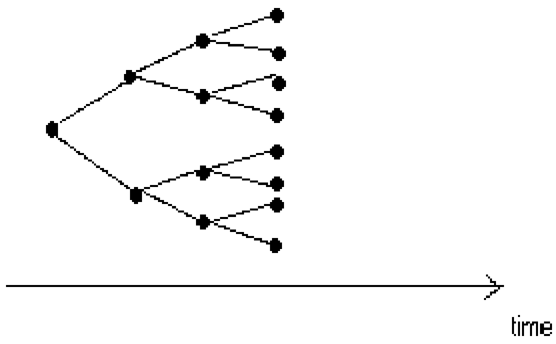
- How does the situation change if the fundamental laws are indeterministic?
  - Albert:
    - It changes a lot, if a theory like GRW is true
  - Following Nino Zanghì's paper (2005):
    - Based on the ideas seen so far, one can construct a <u>unifying approach</u> in which deterministic and indeterministic theories share more commonalities than symmetries

# Indeterminism - 2

- The Ehrenfest model: the indeterministic motion of $N$ balls between two containers
  - Microstate at time t: $X_t = (1, \dots, N)$
    - $1 =$ if the ball is in box 1; 0 otherwise
    - The microstate changes as the balls randomly jump from one box to the other, one at a time
  - How to define the law of evolution of $X$?
    - 1) Probabilistic time-asymmetric way:
      - If initially the state is $X_0$, at $t$ the system will have certain probability to evolve in one of $2^N$ possible states.
    - Problem: the past is essentially different from the future

# Indeterminism - 3

- How to define the law?
  - 2) <u>Graphs of Worldlines</u>:
    - At $t = 0$, the system is in $X_0$
    - At $t = 1$, there are $N$ possible states
    - At $t = 2$, there are $N^2$ possible states
    - …
    - At $t = n$, there are $N^n$ possible states.




  - Thus, the balls distribution evolves in time as described by a graph
  - The set of all possible worldlines of the world for all the $2^N$ possible initial microstates is the <u>union</u> of the graphs corresponding to the different initial microstates (which thus substitutes phase space).

# Indeterminism - 4

- The indeterministic dynamics will determine as typicality measure the one which is time-translation invariant on the space of possible worldlines.

- How to define time-translation invariance in this context?
  - Since no time is privileged, a possible history is a sequence of instantaneous sates with both ends open: $X = [\dots, X_{-1}, X_0, X_1, \dots]$.
  - Time translation of $X : T(X) = [\dots, X_0, X_1, X_2, \dots]$
    - That is, a history in which the state at time $t = -1$ is identical with the state at time $t = 0$ of the original sequence; the state at time $t = 0$ is identical with the state at $t = 1$ of the original sequence; and so on
  - A measure $\mu$ is time-translation invariant if it assigns the same size to the set of possible histories $A$ and its time-translated counterpart $A_{-t}$: $\mu(A) = \mu(A_{-t})$

# Indeterminism - 5

- How does this apply to the GRW theory?
  - The situation is more complicated
  - However, it seems that given the initial wave function, one can draw the set of possible accessible states at a later time and this will be a graph as well.
  - Similarly, therefore, the space of possible microstates is the union of these graphs, one for each initial possible microstate.
  - Hence, the typicality measure is the TT- measure on this space, as defined by the dynamics.

# Indeterminism - 6

- If so, there are more <u>similarities</u> than differences between deterministic and indeterministic theories:
  - Deterministic theories:
    - given an initial state, the dynamical laws determine the typicality measure on the space of possible histories of the world, namely phase space,
  - Indeterministic theories:
    - given an initial state, the dynamical laws determine the typicality measure on the space of possible histories, namely the union of the graphs.
- Thus, in both deterministic and indeterministic cases, given an initial condition and the dynamics, 'everything follows', namely:
  - The <u>space of possible states</u> as well as the <u>typicality measure</u>.

# Conclusions

- A- Albert's account of Boltzmann's necessary ingredients for deterministic theories (modulo PH):
  - Laws of motion + SP (statistical postulate)
- B- Albert's account of Boltzmann's necessary ingredients for GRW theories (modulo PH):
  - Laws of motion
- In this paper I have argued (following Shelly & co.) that:
  - 1) Assuming SP, **typicality**, rather than probability, is enough for explanation
  - 2) SP, as a postulate about the typicality measure, **comes from the dynamics** under symmetry constraints (stationarity, generality).
  - 3) Both indeterministic and deterministic theory can ground a typicality-based explanations since they both require (modulo PH) **only the specification of the laws of motion**.
- If so, there <u>cannot be an fundamental difference</u> between A and B above.
- Therefore, there <u>is no reason</u> in this context to favor GRW to deterministic alternatives

# Thank you for your attention!