Collective phenomena in large interacting ecosystems

Giulio Biroli
Ecole Normale Supérieure

Joint works with G. Bunin (Technion), C. Cammarota (King's College London), V. Ros (ENS), F. Roy (ENS)

Special thanks to Daniel for many useful discussions

Département de Physique
École normale supérieure

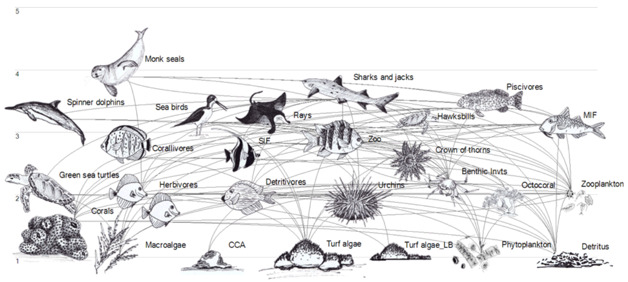
SIMONS FOUNDATION

Ecosystems

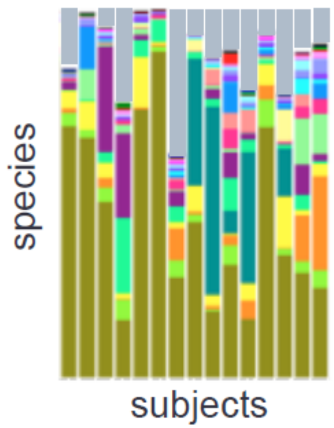
- Communities formed by individuals belonging to different species.
- Interactions between individuals intra and inter species.
- Competition for resources--Cooperation.
- Abundances of species vary dynamically due to the births and deaths.

“Modern” ecosystems (human microbiome)

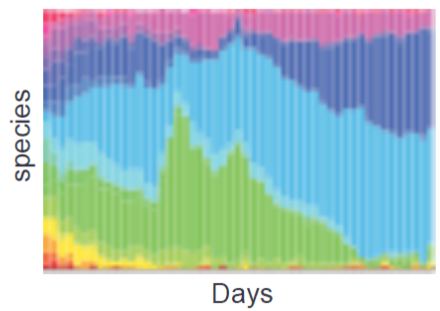
Traditional ecosystem



Abundance profiles



Time dependence



Questions & Motivations

From a few species to <u>many</u> $\longrightarrow$ STATISTICAL PHYSICS

Generic and emergent phenomena of ecosystems

- How many equilibria for the same ecosystem?
Bashan et al. Nature 2016

- How much ecosystems are susceptible to perturbations?
Dethlefsen, Relman PNAS 2011

- Equilibria or chaotic dynamics?
Beninca et al Nature 2008

- What are the factors determining diversity (number of surviving species)?

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left(r_i(K_i - N_i) - \sum_{j(j \neq i)} \alpha_{ij} N_j \right) + \lambda_i$$

$$i = 1, \ldots, S$$

$N_i \geq 0$ abundance of species i
S is the number of species

Large number of species
S larger than 50-100

Well-mixed population: no-space dependence

Dynamics due to intra-and inter-species interactions

Properties of the community reached dynamically?

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left(r_i(K_i - N_i) - \sum_{j(j \neq i)} \alpha_{ij} N_j \right) + \lambda_i$$

$$i = 1, \ldots, S$$

N_i abundance of species i

S is the number of species

Main assumption: complex -> random

(May in ecology & Wigner in physics)

(Determining interactions network: a key inference problem)

α_{ij} Gaussian RVs i.i.d.

$$\langle \alpha_{ij} \rangle = \frac{\mu}{S} \qquad \langle \alpha_{ij}^2 \rangle_c = \frac{\sigma^2}{S}$$

$$\langle \alpha_{ij} \alpha_{ji} \rangle_c = \gamma \langle \alpha_{ij}^2 \rangle_c$$

$\gamma = 1$ symmetric $\qquad -1 \leq \gamma \leq 1$

Small immigration rate and no heterogeneity (Ki=ri=1)

Representative model, see Barbier et al. PNAS 2018

Dynamical Mean Field Theory

One species usual part

Direct effect of interactions

Feedback loop from interactions

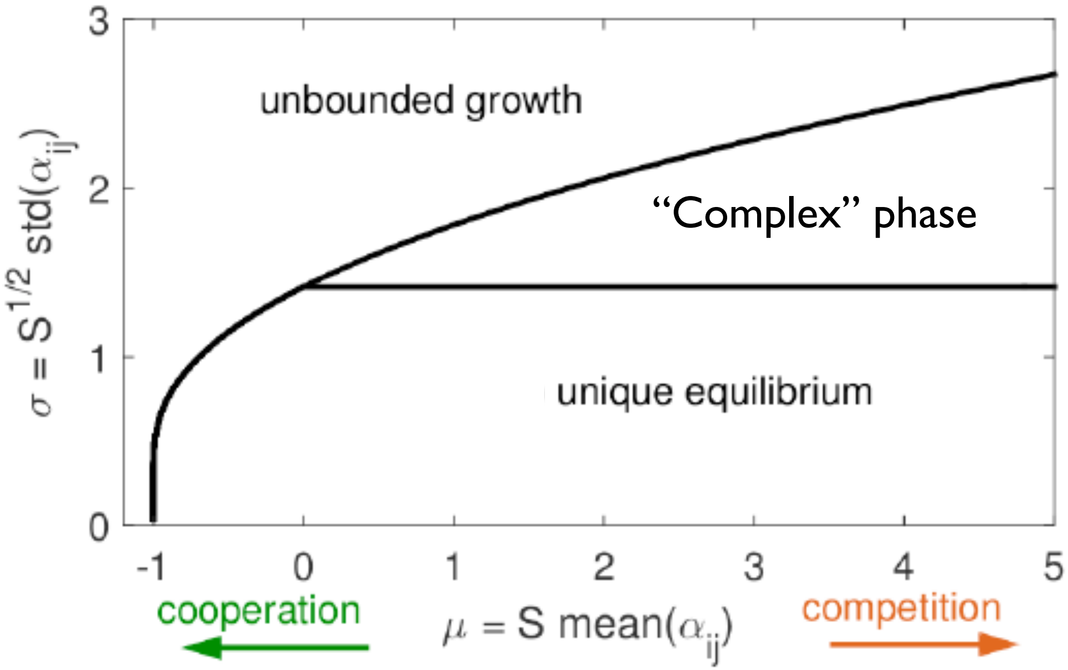
Auxiliary field to define the response

$$\dot{N} = N\{1 - N - \mu m(t) - \sigma\eta(t) - \gamma\sigma^2 \int_0^t K(t,s)N(s)ds - h(t)\}$$

- the **average** $m(t) = \langle N(t) \rangle$;
- the **correlator** $C(t,t') = \langle \eta(t)\eta(t') \rangle = \langle N(t)N(t') \rangle$;
- the **averaged response function** $K(t,s) = \langle \frac{dN(t)}{dh(s)} \rangle$.

Similar to DMFT for spin-glasses: Sompolinsky, Zippelius (1981); Ben Arous, Dembo, Guionnet (2000)

Ecosystems Phase Diagram



Similar for symmetric and non symmetric interactions (here $\gamma = 0$)

Related works and phase diagrams
Sompolinsky, Crisanti, Sommers '88 ; Diederich, Opper '89; Fisher, Mehta '14; Kessler, Shnerb '15; Bunin '16

G. B., G. Bunin and C. Cammarota New J. Phys. 2018 and works to appear with also F. Roy, V. Ros)

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left(r_i(K_i - N_i) - \sum_{j(j \neq i)} \alpha_{ij} N_j \right) + \lambda_i$$

$$i = 1, \ldots, S$$

N_i abundance of species i

S is the number of species

Main assumption: complex -> random

(May in ecology & Wigner in physics)

(Determining interactions network: a key inference problem)

α_{ij} Gaussian RVs i.i.d.

$$\langle \alpha_{ij} \rangle = \frac{\mu}{S} \qquad \langle \alpha_{ij}^2 \rangle_c = \frac{\sigma^2}{S}$$

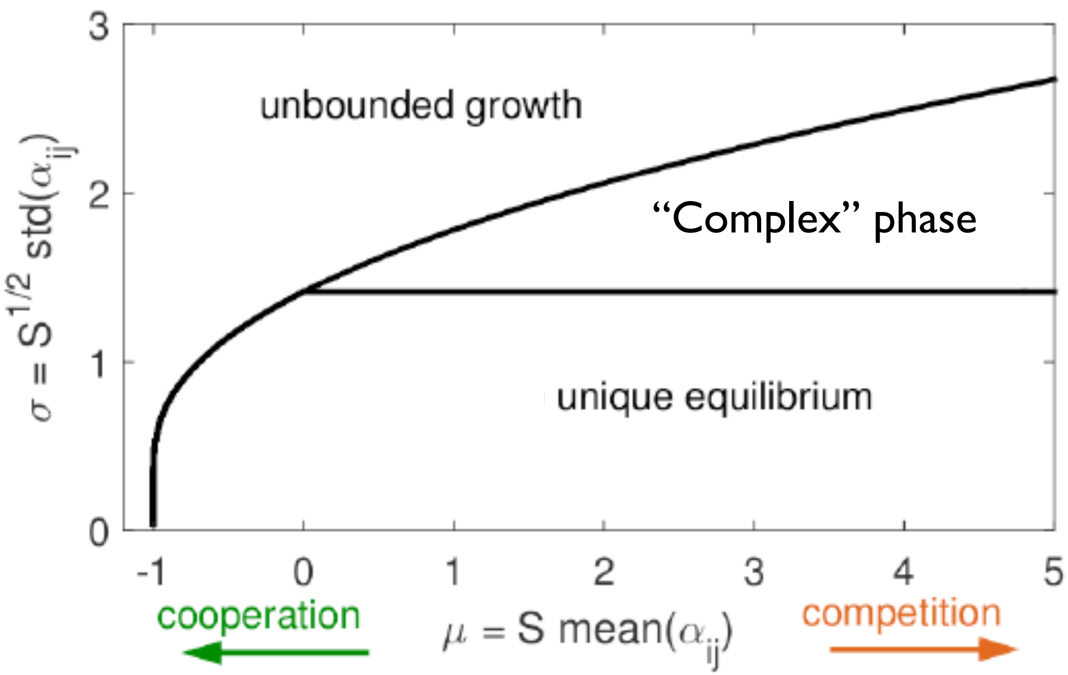
$$\langle \alpha_{ij} \alpha_{ji} \rangle_c = \gamma \langle \alpha_{ij}^2 \rangle_c$$

$\gamma = 1$ symmetric $\qquad -1 \leq \gamma \leq 1$

Small immigration rate and
no heterogeneity (Ki=ri=1)

Representative model, see
Barbier et al. PNAS 2018

Ecosystems Phase Diagram



Similar for symmetric and non symmetric interactions (here $\gamma = 0$)

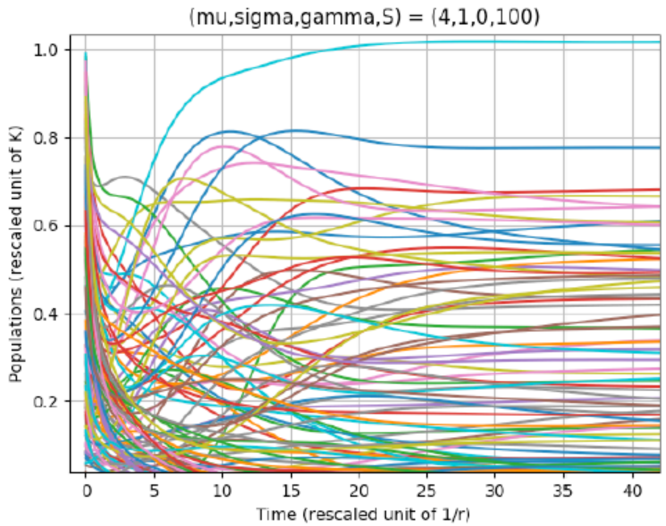
Related works and phase diagrams
Sompolinsky, Crisanti, Sommers '88 ; Diederich, Opper '89; Fisher, Mehta '14; Kessler, Shnerb '15; Bunin '16

[G. B., G. Bunin and C. Cammarota New J. Phys. 2018 and works to appear with also F. Roy, V. Ros)]

Unique Equilibrium Phase

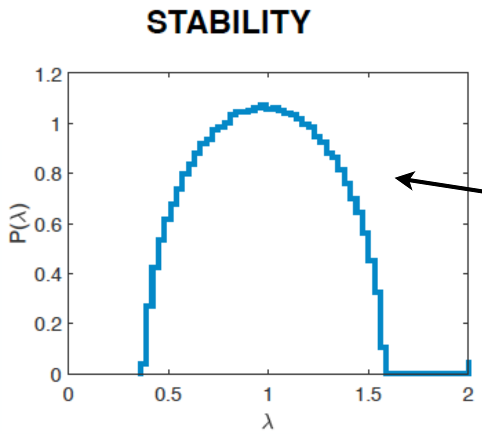
σ Small

Unique equilibrium reached dynamically



Analytics:

- Distribution of the abundances
- Response to perturbation
- Stability of the equilibrium



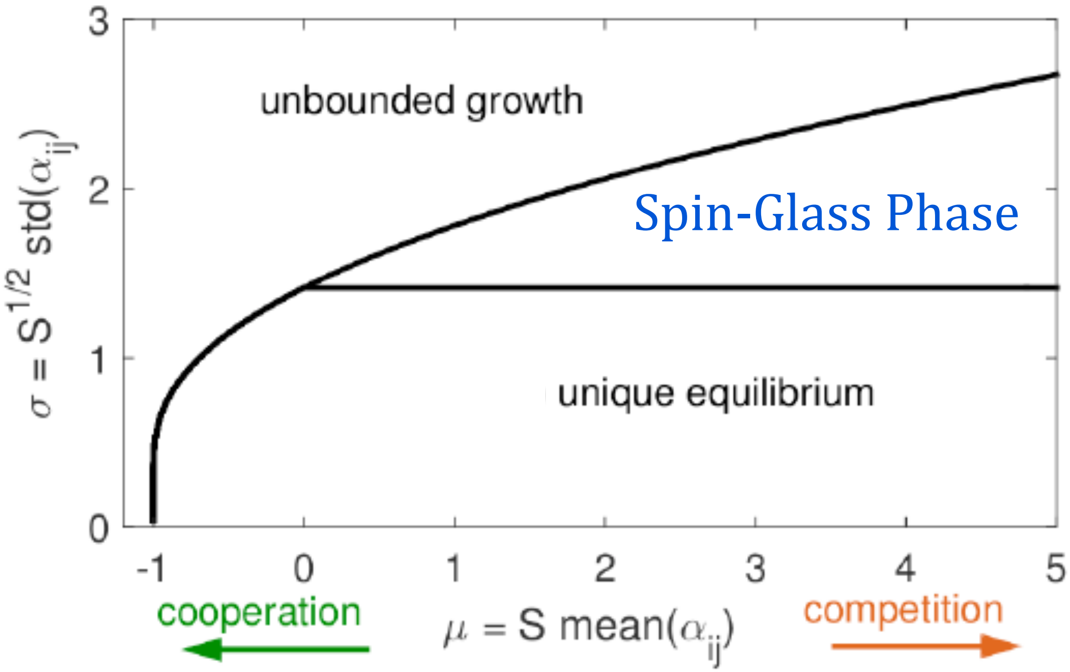
Inverse eigenvalue distribution of the stability matrix

$$\frac{\partial N_i^*}{\partial K_j}$$

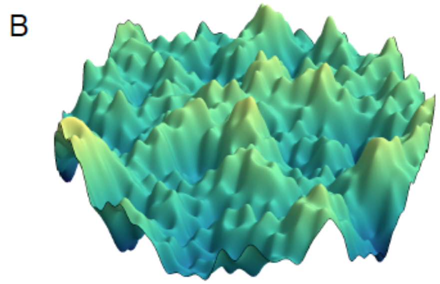
$\sigma = \sigma_c(\gamma)$ Limit of Stability of the one equilibrium phase & phase transition

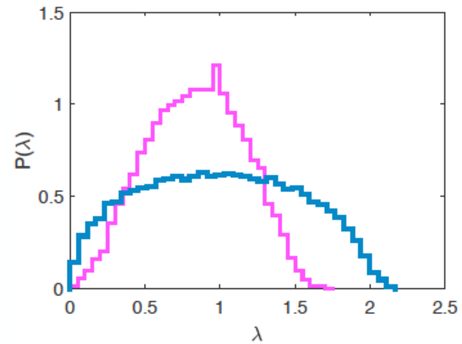
Symmetric interactions

- Descent in a disordered energy landscape
  (like quenches in disordered magnets)



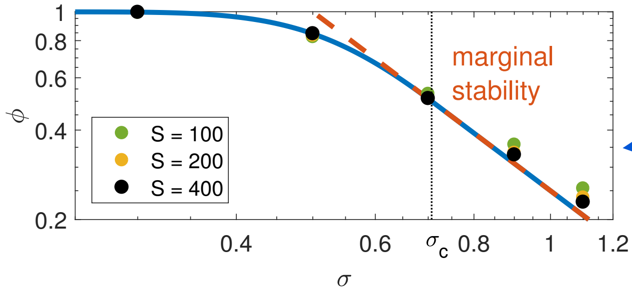
Multiple Equilibria





All equilibria
marginally stable

Marginal stability fixes dynamically
the number of surviving species
(see also Sole, Alonso, McKane 2002)



May's bound

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left(r_i (K_i - N_i) - \sum_{j(j \neq i)} \alpha_{ij} N_j \right) + \lambda_i$$

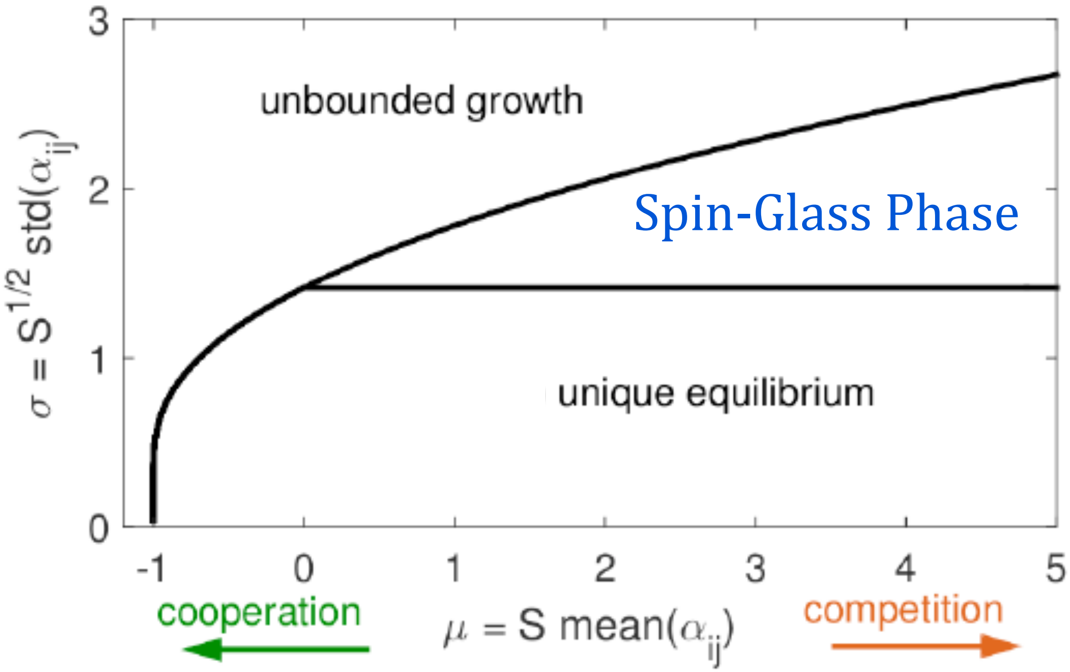
Symmetric α_{ij}

$$\frac{dN_i}{dt} = -N_i \nabla_{N_i} E(\{N_i\})$$

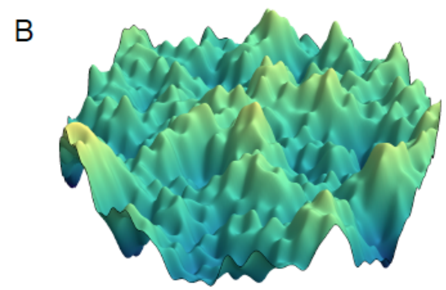
Generalized gradient descent in a disordered energy landscape

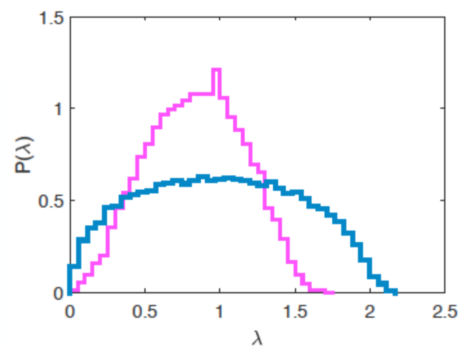
Symmetric interactions

- Descent in a disordered energy landscape
  (like quenches in disordered magnets)

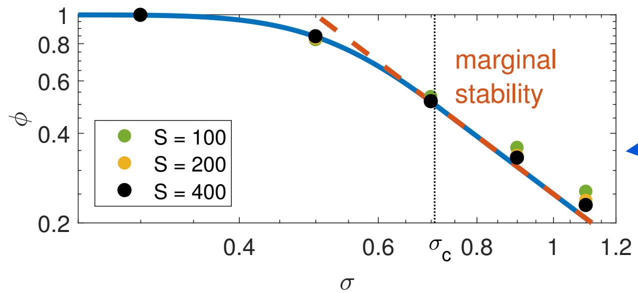


Multiple Equilibria





All equilibria marginally stable



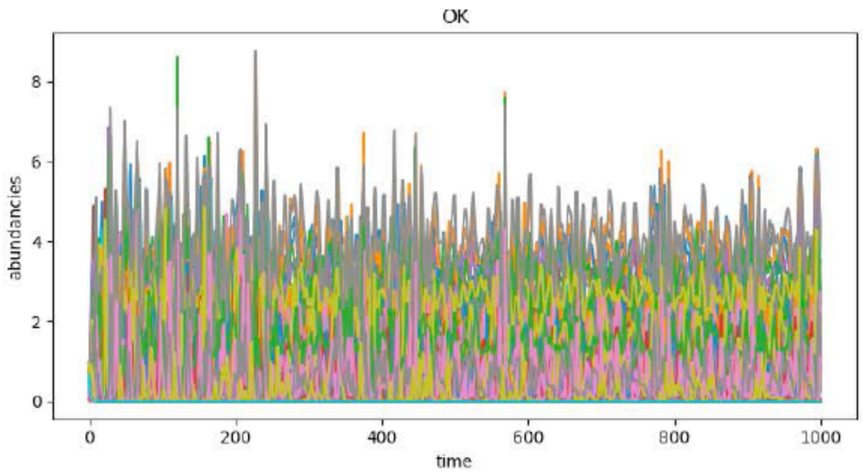
May's bound

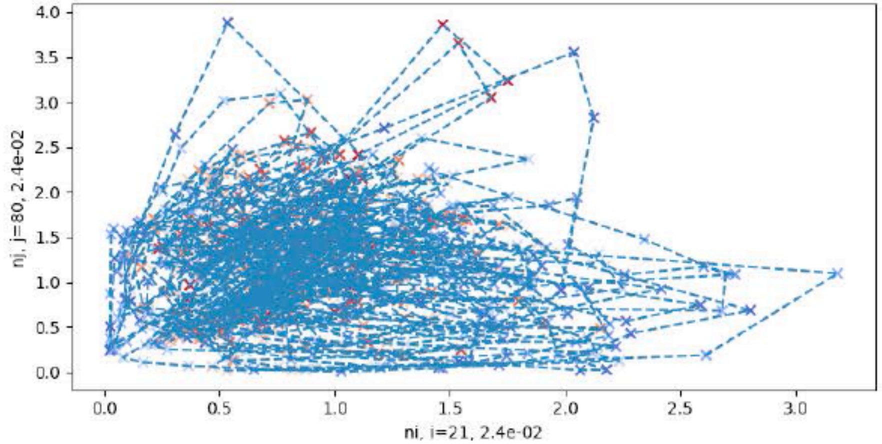
Marginal stability fixes dynamically the number of surviving species
(see also Sole, Alonso, McKane 2002)

Non-Symmetric interactions: chaos

$\gamma < 1$

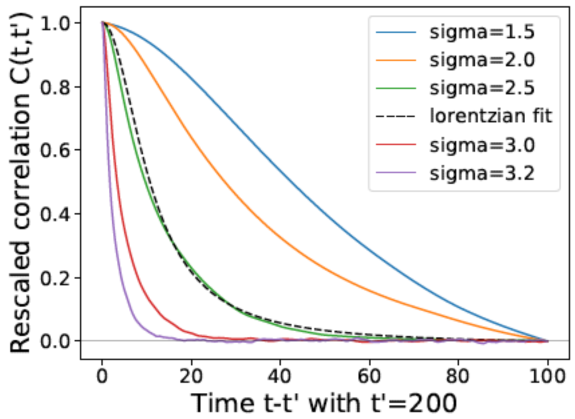
Numerics

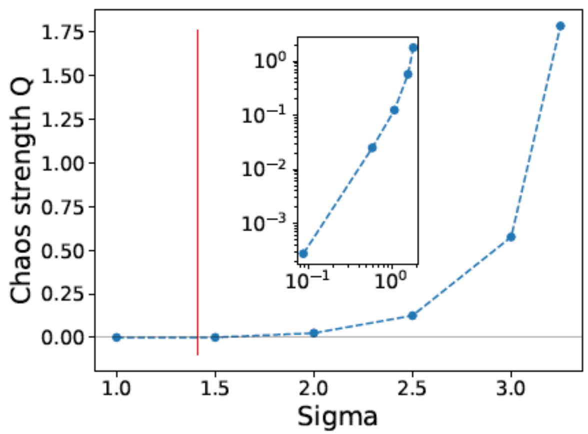


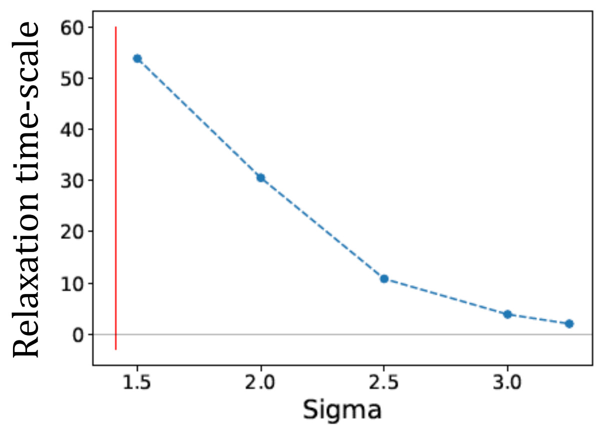


DMFT

transition to chaos: 2nd order out of equilibrium transition







Related works: Sompolinsky, Crisanti, Sommers ’88 ; Kessler, Shnerb ’15

Collective phenomena in large interacting ecosystems

•Different phases of ecosystems from the exact solution of the Lotka-Volterra model of ecosystems

Many perspectives and ongoing works:

- Is chaotic sustainable without immigration?
- Are multiple equilibria always at the May's stability limit (marginally stable)?
- Responses to perturbations, avalanches (extinction cascades)
- Retardation effects, space dependence, evolution…