ETH Without Thermalization

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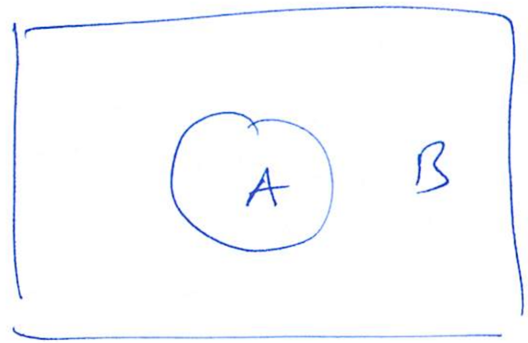
condmat 1406.4863
1811.03114

Outline

1. The representative state problem
2. Intuition and examples
3. Non-equilibrium steady states
4. GKLS Equation with boundary driving
5. Tilted Ising Model - RS/ETH
6. Weak solutions of GKLS Equation

The Representative State (RS) Problem



Given pure state $|AB\rangle$ on $A \cup B$ find pure state $|\psi_A\rangle$ on A alone which can, for practical purposes, reproduce expectation values of typical operators of interest in A.

$|\psi_A\rangle$ is a RS.

Intuition

$\rho_A = \mathrm{Tr}_B |AB\rangle\langle AB|$ contains all needed information

Write $\rho_A = e^{-H_E}$ $(T_E = 1)$

If H_E is generic and obeys ETH then we can reproduce $\mathrm{Tr}(O e^{-H_E})$ by $\langle \alpha | O | \alpha \rangle$ where α is a suitably chosen eigenstate of H_E.

Recall $\langle \alpha | \hat{A} | \alpha \rangle = O(E) + e^{-S(E)/2} f(E) R_\alpha$

Examples

1. Randomly picked states

(Grover, Fisher)

$$|AB\rangle = \frac{1}{\sqrt{D_{A\cup B}}} \sum_{C_{AB}} \mathrm{sgn}(C_{AB})\, |C_{AB}\rangle$$

$D_{A\cup B} \to 2^{N_A} \cdot 2^{N_B}$

$\mathrm{sgn}(C_{AB})$: random ± 1

$|C_{AB}\rangle \to |C_A\rangle|C_B\rangle$

For $C \subset A \subset A\cup B$, $|C|$ finite & bounded

$$\langle AB|\hat{O}|AB\rangle = \mathrm{Tr}_{\rho}(\hat{O}) + O\left(\frac{2^{N_C}}{\sqrt{2^{N_A+N_B}}}\right) \equiv O\left(\frac{D_C}{\sqrt{D_{A\cup B}}}\right)$$

$\rho = \mathbb{1}$

where $\hat{O}$ lives on C.

RS ?

Pick random state on A $\equiv |\psi_A\rangle$

Then

$$\langle \psi_A | \hat{O} | \psi_A \rangle = \mathrm{Tr}_{\alpha}\, \hat{O} + O\left(\frac{2^{N_C}}{\sqrt{2^{N_A}}}\right)$$

and becomes exact if $|C| \ll |A|$

Generalizes to Haar (measure) random states

States are maximally entangled! (Page)

2. Free fermion states

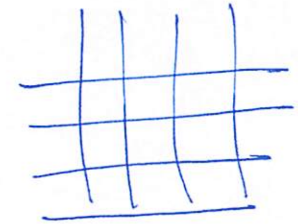
For a FF state, $\rho_A = \frac{e^{-H_E}}{Z}$ where $H_E = \sum_{i=1}^{|A|} \epsilon_i f_i^\dagger f_i$

Can find ES which reproduces $\langle \hat{O} \rangle$ with $\hat{O}$ spread over conserved quantities $f_i^\dagger f_i$.

Prescription: Fill states i with probability ~~&~~ given by Fermi-Dirac distribution with $T_E = 1$, $\mu_E = 0$.

(See 1406.4863 for details)

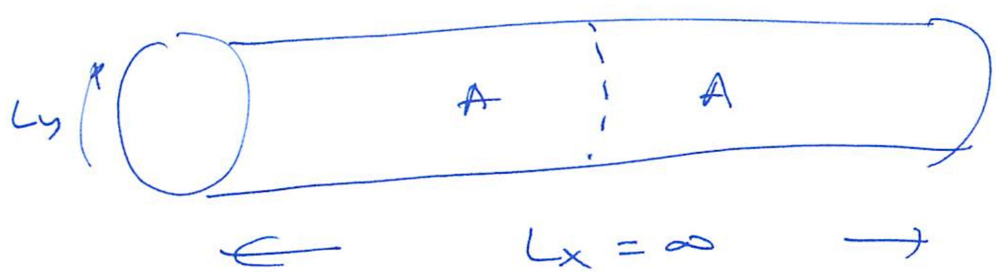
3. RK wavefunction for Ising Model



$$|AB\rangle = \sum_{\sigma} e^{-E_{cl}(\bar{\sigma})/2} \, |\bar{\sigma}\rangle$$

$$\sum_{ij} \beta_x \left(\sigma^z_{i,j}\, \sigma^z_{i,j+1}\right) + \beta_y \left(\sigma^z_{i,j}\, \sigma^z_{i+1,j}\right)$$

$\Rightarrow \quad p(\bar{\sigma}) = e^{-E_{cl}(\bar{\sigma})}$ = Boltzmann wt 4 classical Ising configuration



Classical transfer matrix — Poisson statistics

Quantum HE — GOE

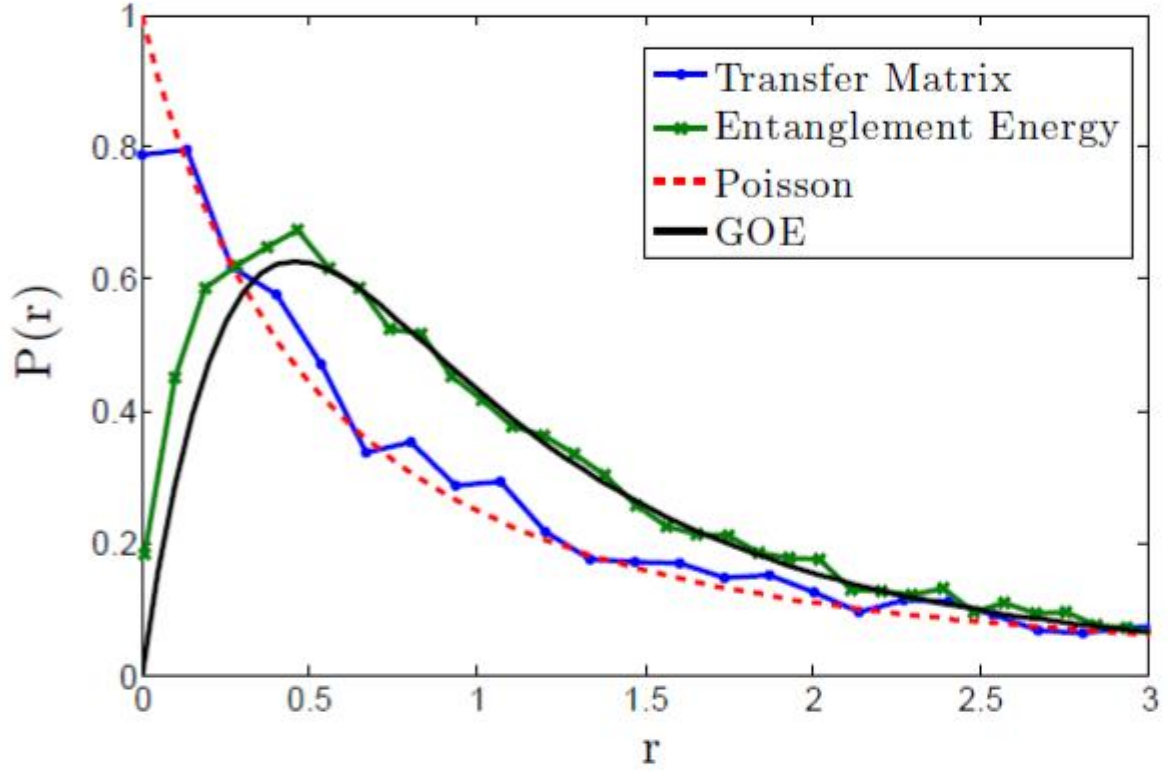
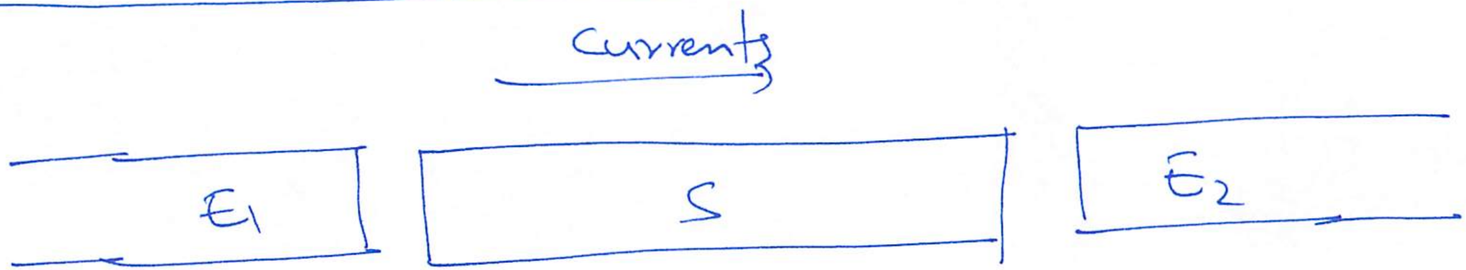


FIG. 2. Level spacing ratio statistics of H_E for the Rokhsar Kivelson state (9) compared to the Poisson and GOE distributions. The statistics clearly look GOE consistent with a non-integrable H_E. This is to be contrasted with the Possoinoin statistics of the integrable transfer matrix T_{σ_i,σ_j} in (11). r refers to the ratio of subsequent level spacings, and $P(r)$ is the probability of obtaining a given r. The GOE form is derived in Ref. 25.

Non-equilibrium steady states (NESS)



Examine $\lim_{t\to\infty}\left(\rho_S(t) = \mathrm{Tr}_E\, |SE\rangle\langle SE|\right) = \rho_{NESS}$

Question: Can we replace ρ_{NESS} by a RS on S alone?

More generally: Does H_{SS} defined by $\rho_{SS} = e^{-H_{SS}}$ obey

NESS-ETH?

Consider specific setting for NESS.

The GKLS Master Equation

History 1710.05993

$$\frac{d\rho}{dt} = \underset{\uparrow \atop \text{Lindbladian}}{\hat{\mathcal{L}}(\rho)} \equiv \underset{\uparrow \atop \text{Liouvillian}}{-i[H,\rho]} + \underset{\uparrow \atop \text{Dissipator}}{\sum_\mu \left(2L_\mu \rho L_\mu^\dagger - \{L_\mu^\dagger L_\mu, \rho\}\right)}$$

$L_\mu \equiv$ jump operators

Various derivations; nice properties (complete positivity)

Consider $d=1$ with L_μ restricted to boundaries:

"boundary driven GKLS" Prosen, Žnidarič e Co

Minimal model for NESS

$$\hat{\mathcal{L}}(\rho_{NESS}) = 0$$

Tilted Ising Model

$$\begin{array}{cccccccc} \circ & \circ & \circ & \circ & \cdots & \circ & \circ \\ 1 & 2 & & & & N-1 & N \end{array}$$

$$H = \sum_{j=1}^{N-1} \left(\sigma^x_j \sigma^x_{j+1} + h \sigma^z_j + g \sigma^x_j \right)$$

non-integrable for generic h, g

GOE absent driving

Only energy current $J = \dfrac{\sum_{j=2}^{N-1} J_j}{N-2} = \dfrac{h \sum_{j=2}^{N-1} \sigma^y_j \left(\sigma^x_{j+1} - \sigma^x_{j-1} \right)}{N-2}$

Two jump operators at site 1 $\quad L_1, \; L_2 = L_1^\dagger$

$N \quad L_3, \; L_4 = L_3^\dagger$

Create thermal gradient across system

Computations: ρ_{NESS} has an efficient MPO representation

Find via TEBD

Diagonalize upto $N = 12$.

Examine level statistics and expectation values.

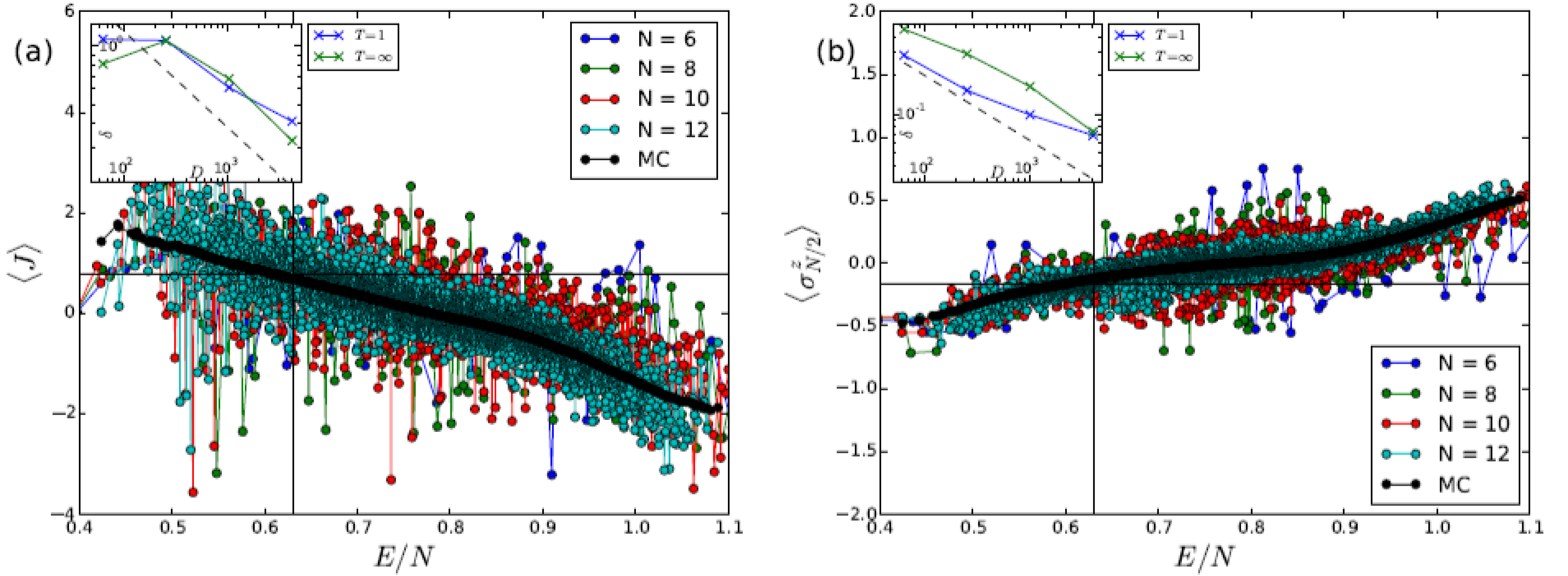
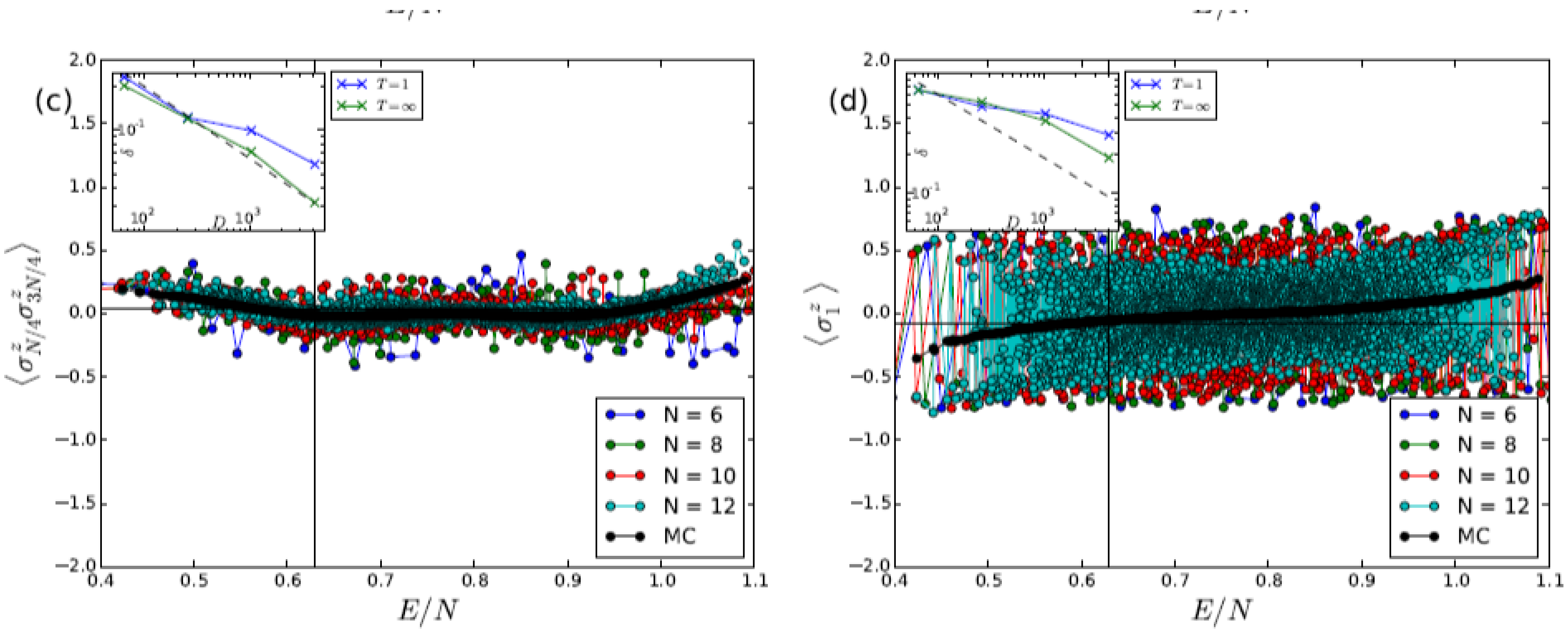


FIG. 2. (Color online) Expectation values and standard deviations of the operators (a) J (b) $\sigma^z_{N/2}$ (c) $\sigma^z_{N/4}\sigma^z_{3N/4}$ (d) σ^z_1 in the eigenstates of H_{ss} as a function of pseudoenergy. (Main) The expectation values appear to converge to a smooth function of energy with increasing system size, which is the "microcanonical" (MC) value computed by averaging over an energy window of $\Delta E = 0.025$. The expectation value in ρ_{ss} and the energy density ϵ^* of the representative state are denoted by the horizontal and vertical lines respectively. (Inset) Standard deviations of the operators as a function of the Hilbert space dimension. The scaling of $1/\sqrt{D}$ is denoted by the black dashed line. The standard deviation appears to scales as $\sim 1/\sqrt{D}$ for eigenstates in the middle of the spectrum ($T = \infty$), consistent with the predictions of NESS-ETH. The standard deviations around the eigenstates around the representative energy ($T = 1$) also decay with increasing Hilbert space dimension, although they show a scaling slower than $1/\sqrt{D}$. The data is shown for the parameters $(g, h, \beta_l, \beta_r) = (0.5, -1.05, 0.1, 1)$.



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Weak Solutions of GKLS Eqn?

Are RS solutions of the GKLS Eqn? More precisely $\rho_{RS} = |RS\rangle\langle RS|$?

Boundary driven GKLS spectra are gapless, Znidarič (2015) so maybe.

Instead claim weak solution, i.e. $\frac{d}{dt}\langle\hat{O}\rangle = \frac{d}{dt}\mathrm{Tr}(\hat{O}\rho_{RS}) = 0$

Need to check $\mathrm{Tr}\left(\frac{d\rho}{dt}\hat{O}\right) = 0 = \langle n| i[H,\hat{O}] + 2L_n O L_n^\dagger - \{L_n^\dagger L_n, \hat{O}\}|n\rangle$

Expect via NESS-ETH. Check directly for $H \equiv \hat{O}$.

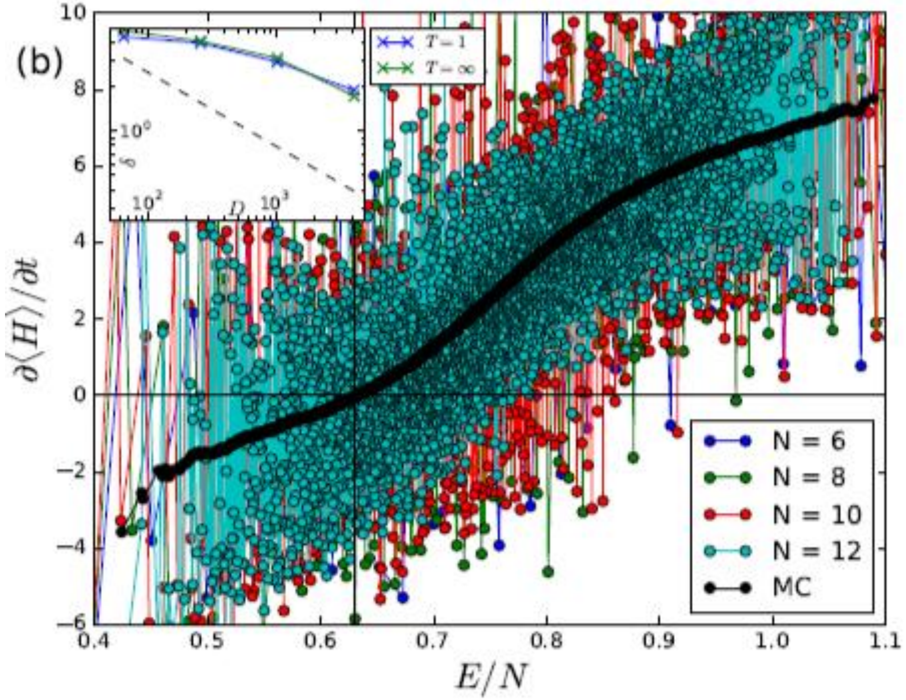

FIG. 3. (Color online) (a) $\langle J/(N-2) - J_{N/2}\rangle$ and (inset) its standard deviation in the eigenstates of ρ_{ss}. (b) $\frac{\mathrm{d}\langle H\rangle}{\mathrm{d}t}$ for $\langle H\rangle$ computed in the eigenstates of ρ_{ss}. Its vanishing at the representative energy shows that $\rho_n = |n\rangle\langle n|$ is a weak solution of the GKLS Master Equation, where $|n\rangle$ is the representative eigenstate. See caption of Fig. 2 for details on the labels and parameters used.

Finally

1. Diagonal ETH can be extended to several settings where there is no thermalization. Leads to RS.

2. Specifically, it can be extended to NESS in boundary driven GKLS Equations.

3. Should check off-diagonal ETH.

4. Is there a formalism that directly yields pure states as NESS ~ "scattering states" w/o leads?