Long-range interactions between bodies in an active fluid

Yariv Kafri
Technion - Israel Institute of Technology

Based on:

Yongjoo Baek (Technion -> Cambridge), **Alex Solon** (MIT -> Paris 6),
Xinpeng Xu (Technion -> GTIIT), Nikolai Nikola (Technion)

PRL,120, 058002 (2018)

Omer Granek (Technion)
with interactions - not discussed

Active Systems

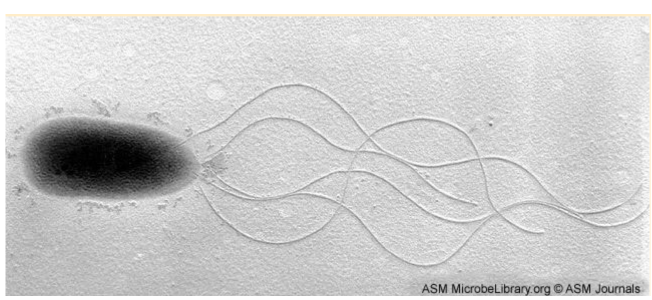
Each particle consumes/dissipates energy to generate motion

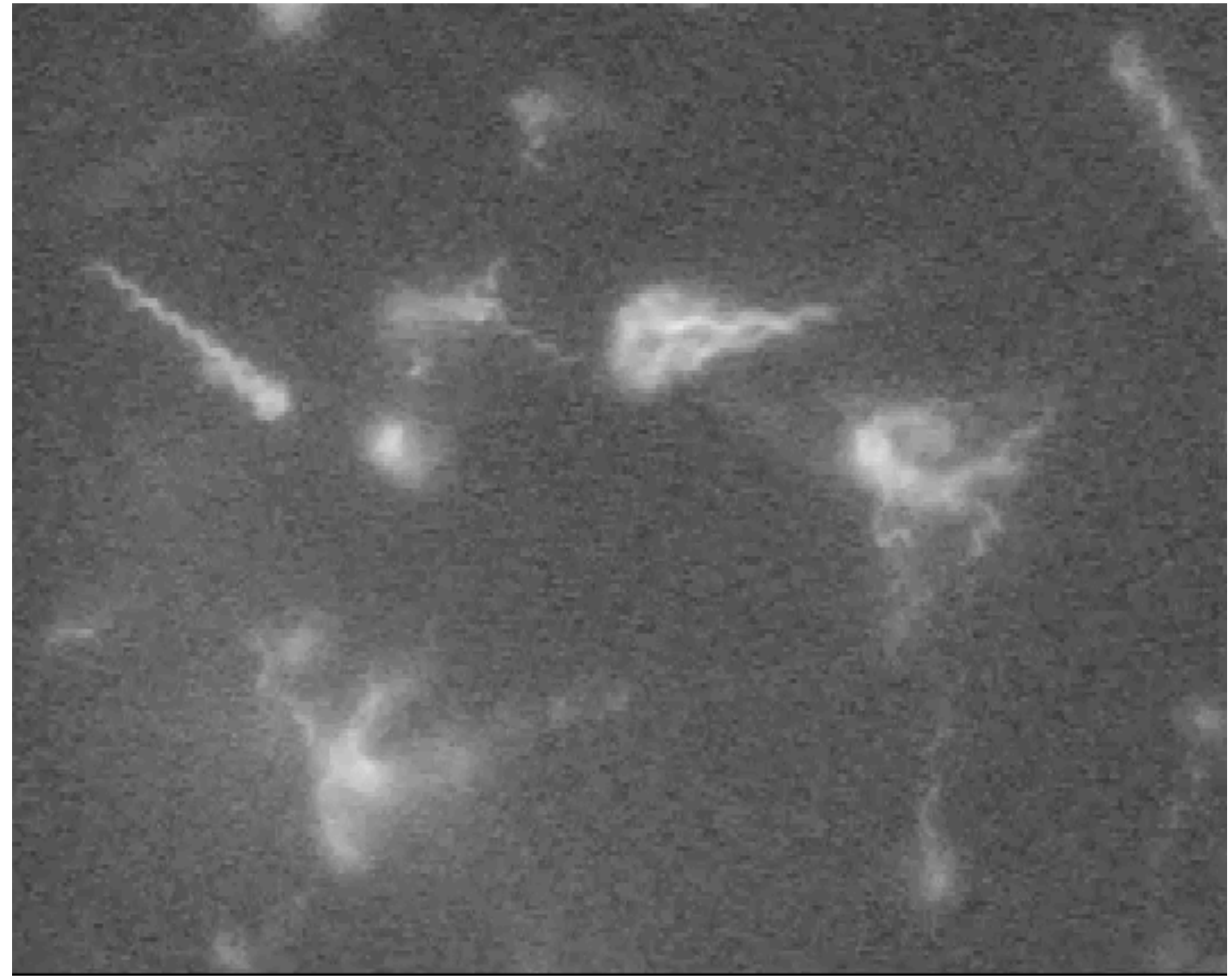
E. Coli

size $3\mu m \times 1\mu m$

velocity $20\mu m/s$

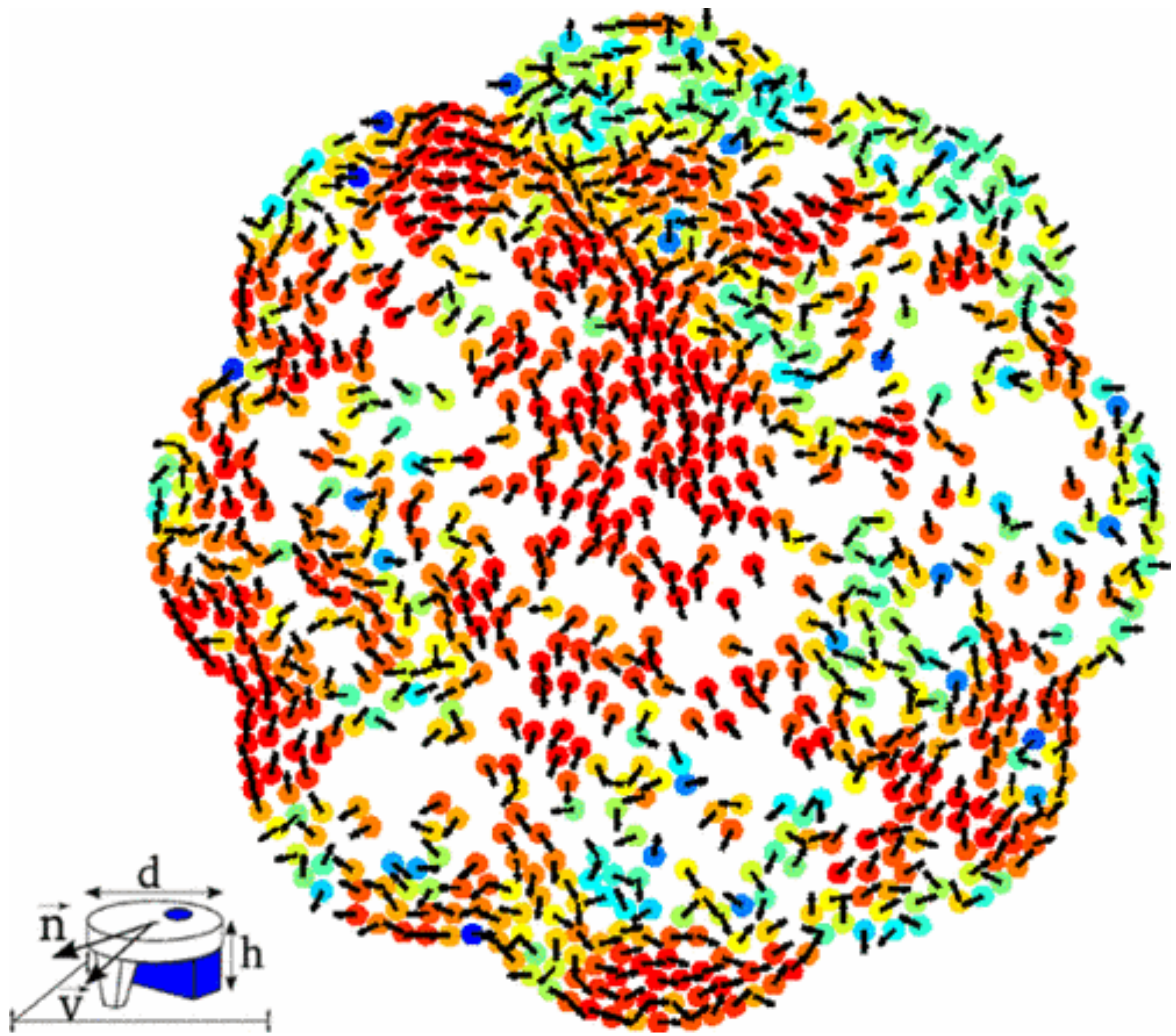
tumble $1/s$





(from Berg lab website)

Shaken Granular Particles

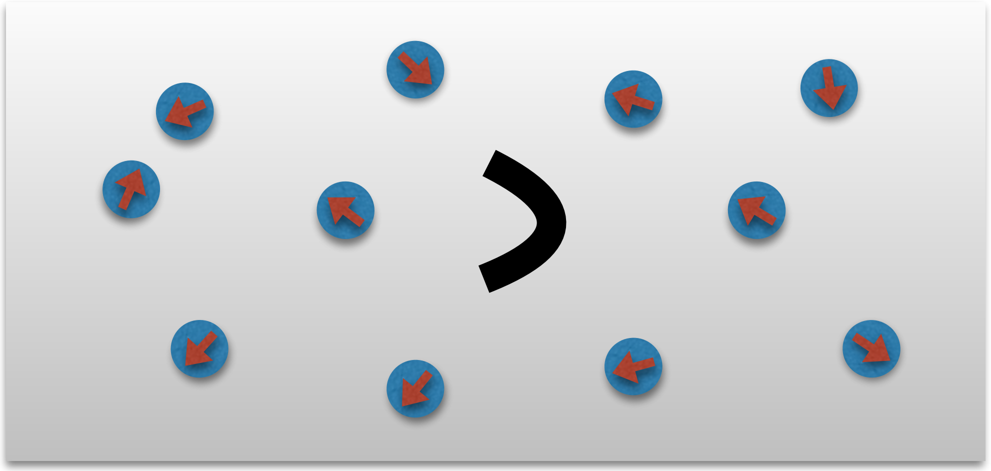


e.g. Dauchot lab

Other example: molecular motors, actin, bird flocks…

Premise of talk:

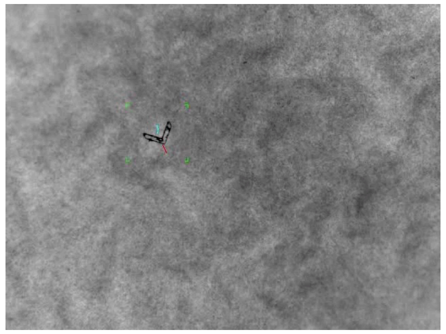
One **asymmetric** body *held* in active bath



Breaking of time reversal symmetry

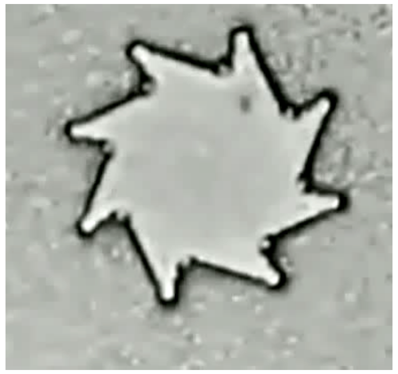
=

Current of active particle + Force on body (related)



[Kaiser *et. al*., PRL **112**, 158101 (2014)]

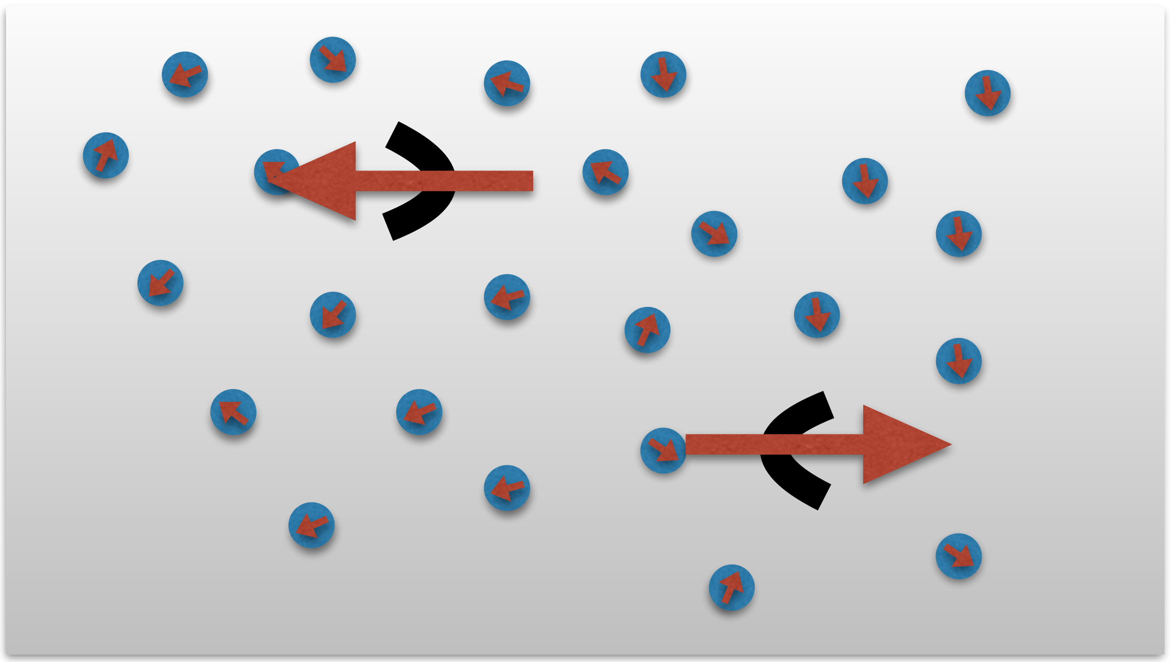
Wedge in a *B. subtilis* suspension



[Di Leonardo *et. al*., PNAS **107**, 9541 (2010)]

E. coli-driven ratchet

Two **asymmetric** bodies *held* in active bath



Each body influences currents and forces on other object

Interactions mediated by active medium

Show and quantify:

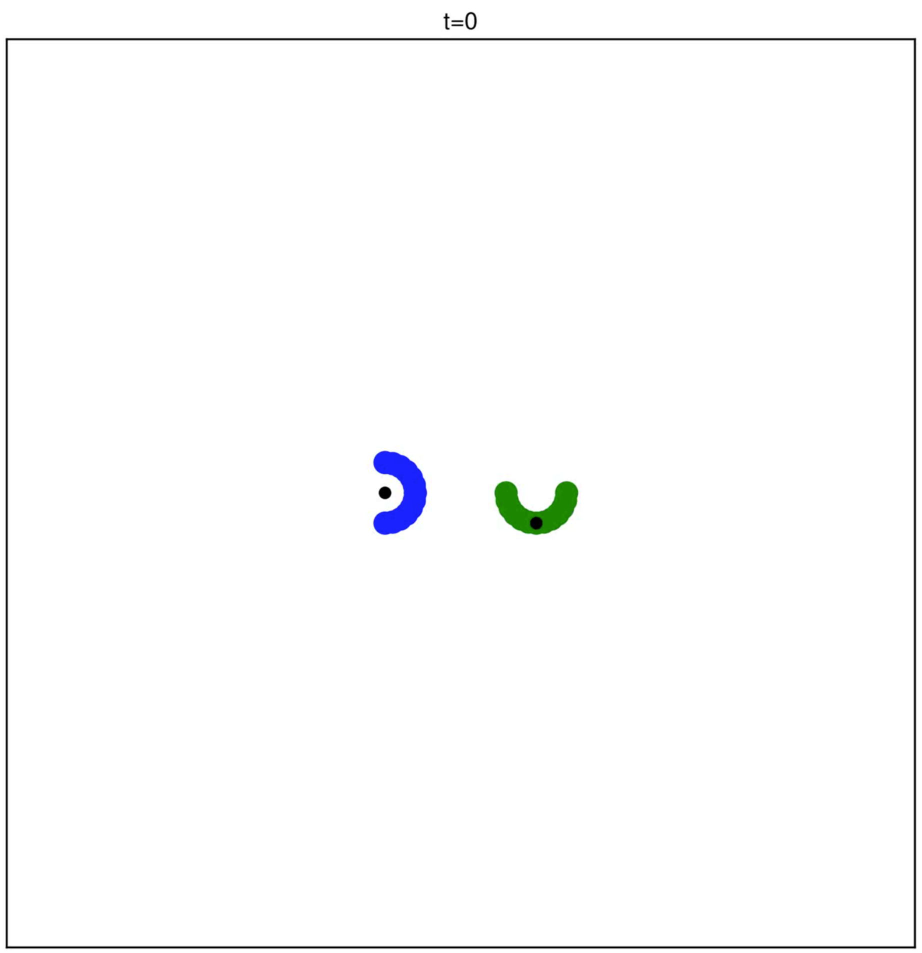
1. **Long-range (power-law)**
2. **Anisotropic**
3. **No action reaction**

Also in <u>dilute systems</u> - not due to interactions in active system

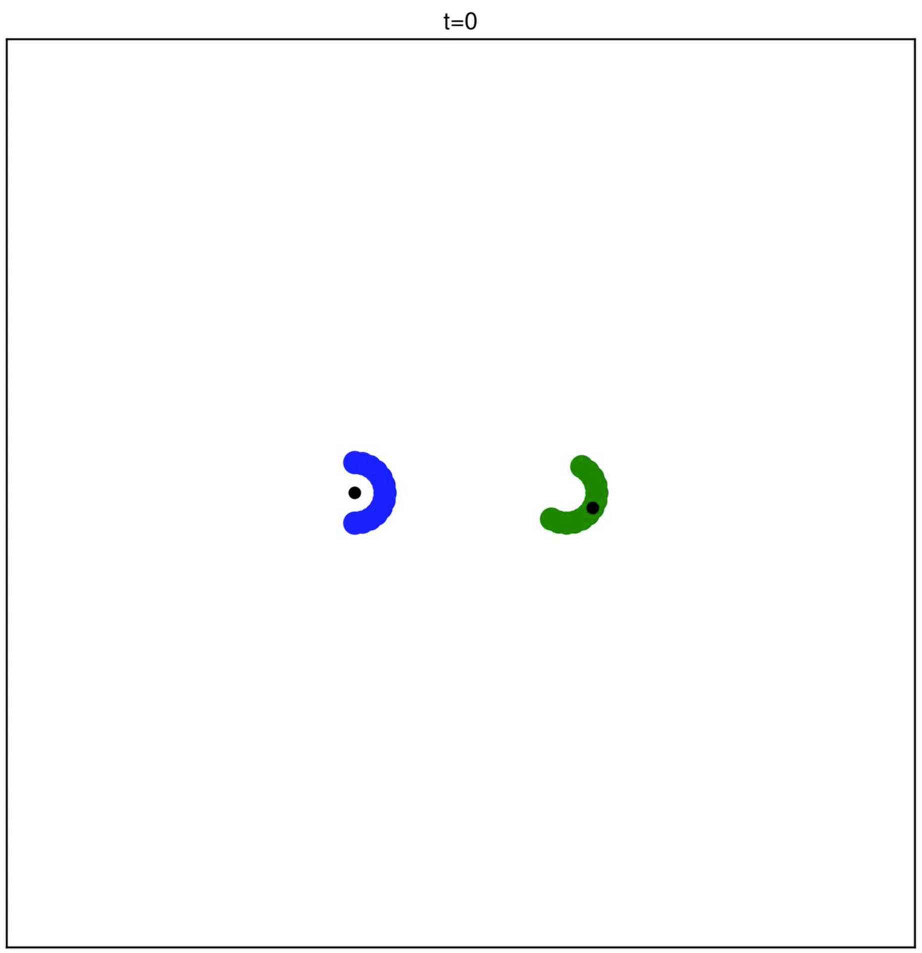
(analytical far field in terms of single body properties, adiabatic limit, non-interacting and pairwise interacting - same results)

Can control interactions by shapes

pinned (tuned)



free (tuned)



(adiabatic limit explains)

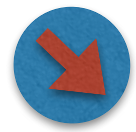
Outline of talk:

- A single body (asymmetric) in an active fluid
  - Forces and currents
  - Far field density and current field

- Two bodies in an active fluid
  - Understand interactions from single body results
  - Derive *forces and torques*

Active Brownian and Run-and-Tumble particles (2d)

(motivations - bacteria, colloids, shaken granular systems..)



- particles with position and orientation in the absence of a potential

$$\dot{\mathbf{r}} = v\mathbf{e}_\theta$$

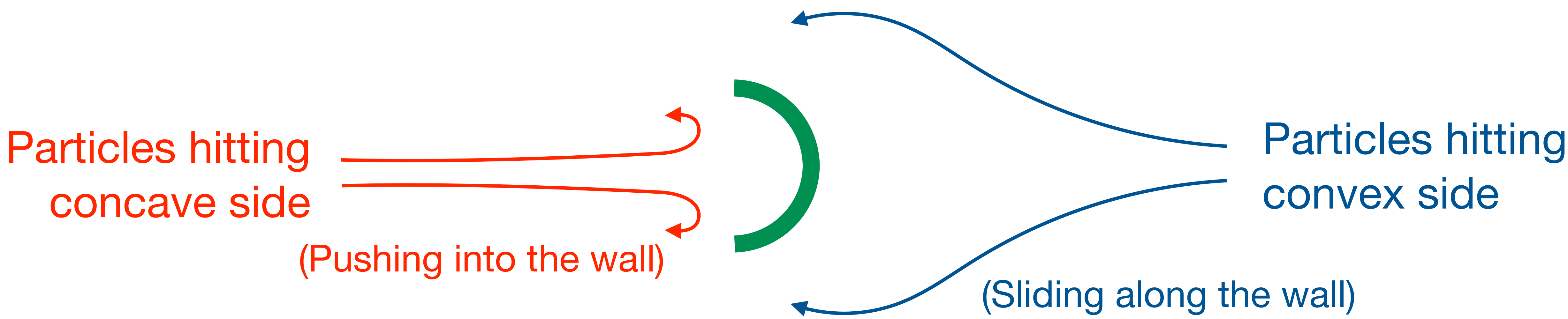
$$\begin{cases} \text{Randomly change } \theta \text{ at rate } \alpha \\ \dot{\theta} = \sqrt{2D_r}\,\eta_r(t) \end{cases}$$

White noise of unit variance

Run-and-tumble particles

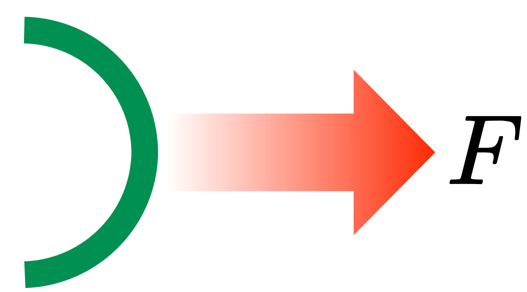
Active Brownian particles

Single body in an active fluid



(Galajda et. al. J. Bacteriol 2007)

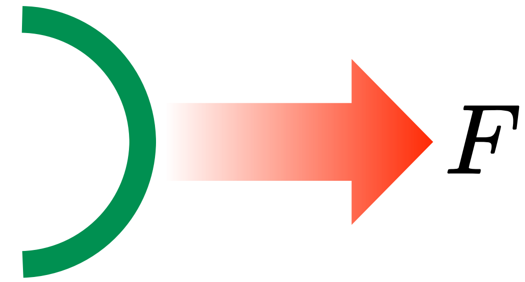
An active fluid applies a nonzero net force
on an asymmetric body (depends on potential and curvature).



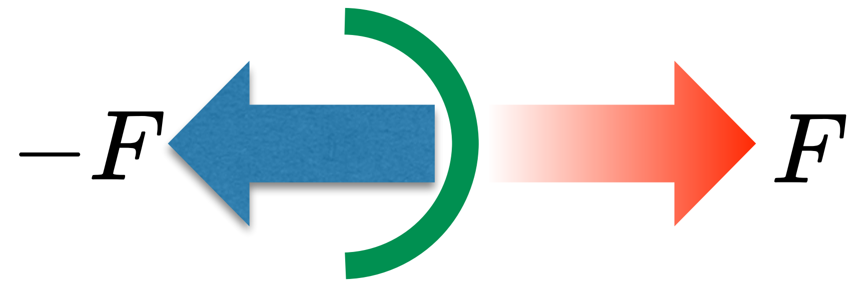
$$F = \int d^2 r \rho(\mathbf{r}) \nabla V(\mathbf{r})$$

body

An active fluid applies a nonzero net force
on an asymmetric body.



Body applies a nonzero net force
on the particles.



Force on particles generate flows

Can show 1:

PRL 2016
with Nikolai,Solon,Kardar,Tailleur

For a general object (also with pairwise interactions)

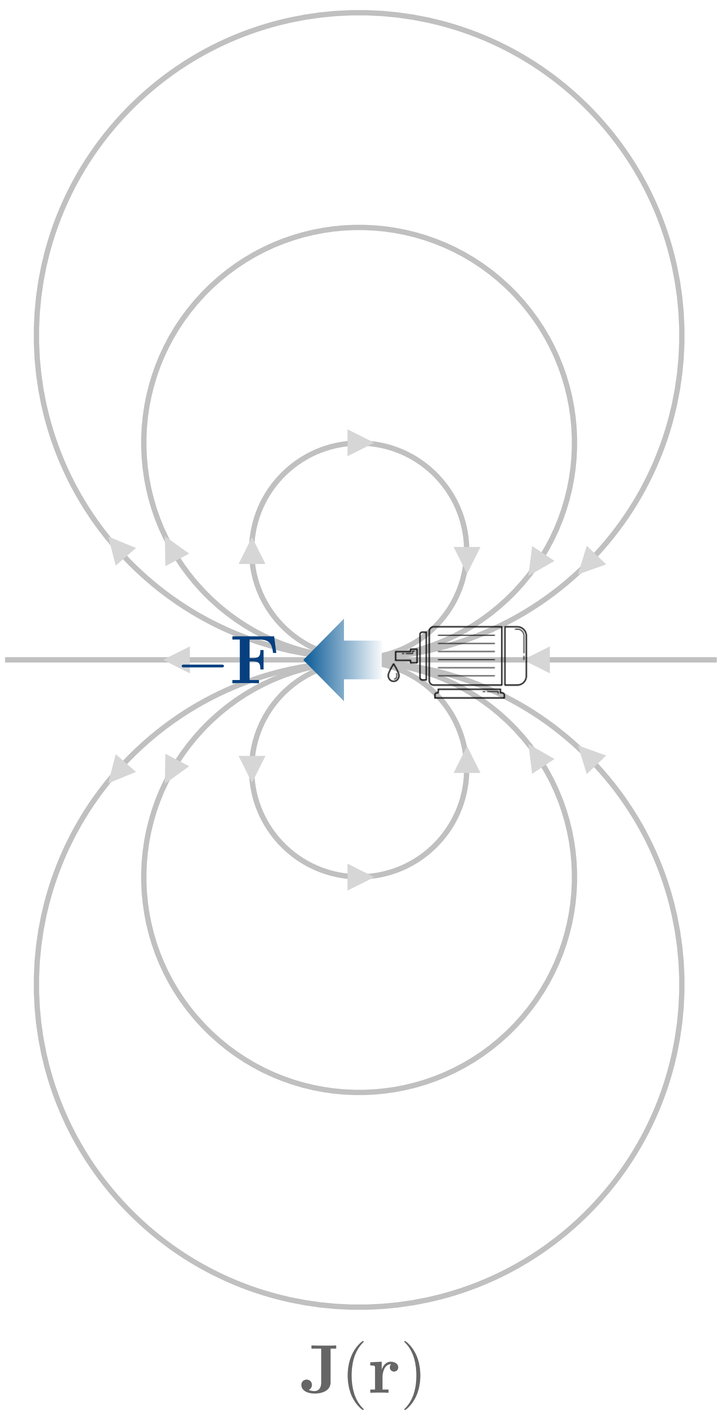
$$\mu F = -\int d^2 r J(\mathbf{r})$$

current density

Note - Force and current depend on potential/shape

Can show 2: (dilute system + pairwise)

In *far field* **asymmetric object acts like a pump in diffusive medium**



Steady-state equation

$$\boldsymbol{\nabla}\cdot\mathbf{J} = -D_{\text{eff}}\nabla^2\rho - \boldsymbol{\nabla}\cdot[\mu\mathbf{F}\delta(\mathbf{r})] = 0$$

Point force (≒ 2D dipole)

Fluid density

$$\rho(\mathbf{r}) = -\frac{\mu}{2\pi D_{\text{eff}}}\frac{\mathbf{r}\cdot\mathbf{F}}{r^2}$$

Diffusive current

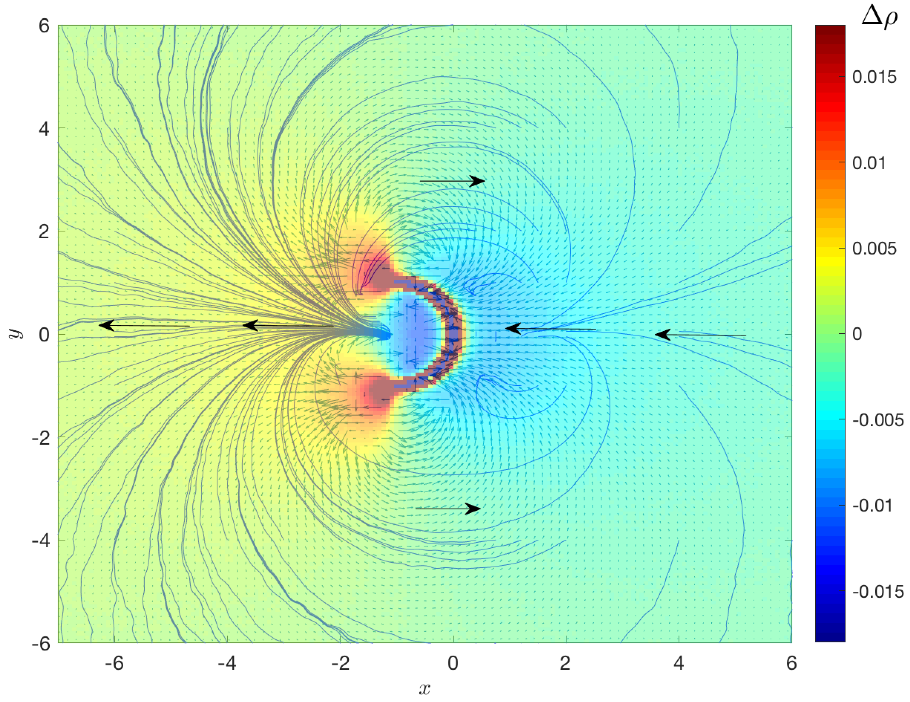
$$\mathbf{J}(\mathbf{r}) = \frac{\mu}{2\pi}\left[\frac{\mathbf{F}}{r^2} - \frac{2(\mathbf{r}\cdot\mathbf{F})\mathbf{r}}{r^4}\right]$$

long-range density and current fields
(non-local distribution function)

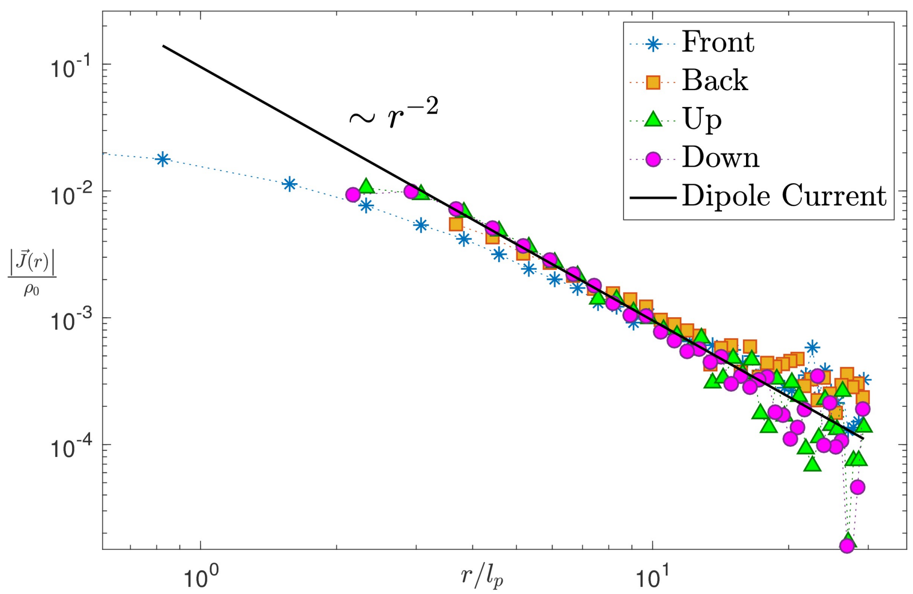
F - **force acting on body**

Similar to driven bond - Sadhu, Majumdar, Mukamel PRE 84, 051136, 2011

Density and current streamlines



Current along axes



Recap:

A single body (asymmetric) in an active fluid

- Experiences a force
- Generate long range density modulations and currents
- density and profile characterised by a **single measurable quantity** (at this order)

***Comment:* Can also get perturbative (in body potential + small run length) expressions - leading order** $l_r^2 (\nabla V)^3$

$$\mathbf{p} = -\mathbf{F}$$ **dipole moment**

$$\frac{1}{\rho_b l_r^2}\left(\frac{D_{\text{eff}}}{\mu}\right)^2 \mathbf{p} \simeq c_1 \int d^2\mathbf{r}\,(\boldsymbol{\nabla} V)^2 \boldsymbol{\nabla} V + c_2 \int d^2\mathbf{r}\,(\boldsymbol{\nabla} V) \int d^2\mathbf{r}'\,\frac{(\boldsymbol{\nabla}' V)^2}{|\mathbf{r}-\mathbf{r}'|^2}$$
$$+ c_3 \int d^2\mathbf{r}\,(\boldsymbol{\nabla} V) \int d^2\mathbf{r}'\,\frac{[(\mathbf{r}-\mathbf{r}')\cdot(\boldsymbol{\nabla}' V)]^2}{|\mathbf{r}-\mathbf{r}'|^4}$$

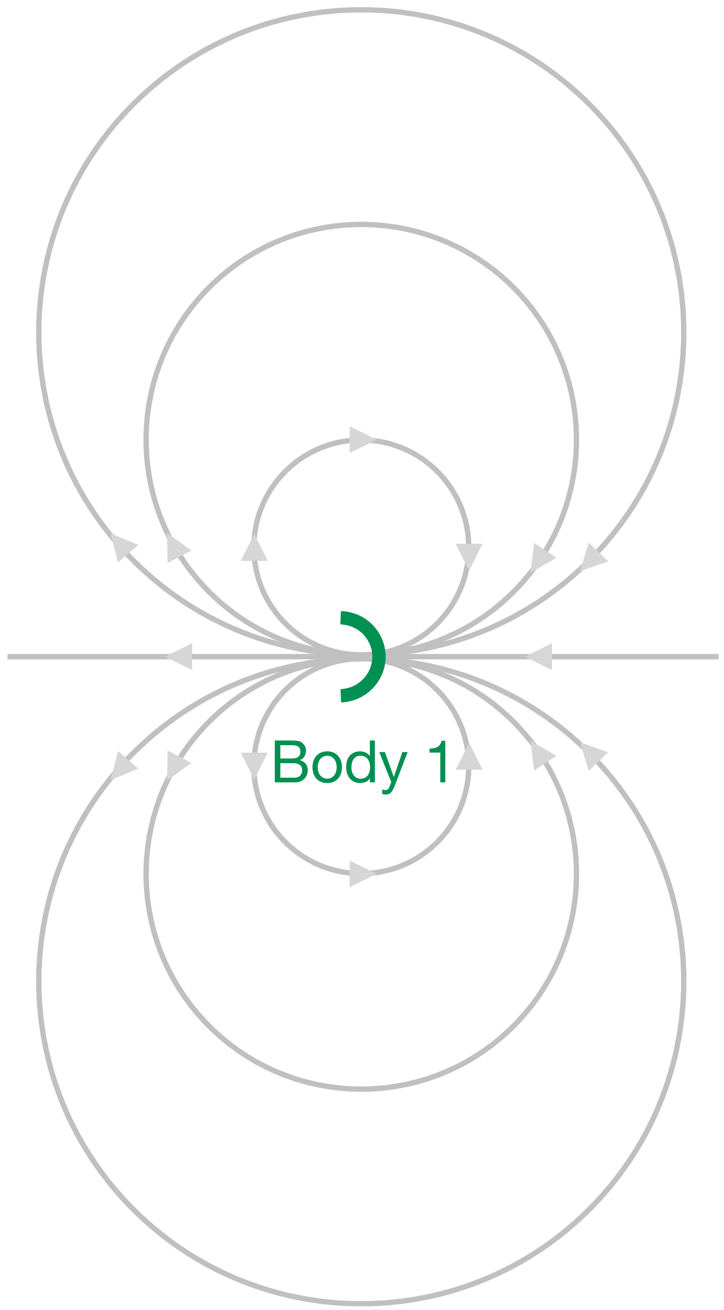
$c_j(\alpha, D_r)$ Known functions

Note - non-local function of potential!

Two bodies in an active fluid: characterized by

$$\mathbf{F}_1 = -\mathbf{p}_1$$

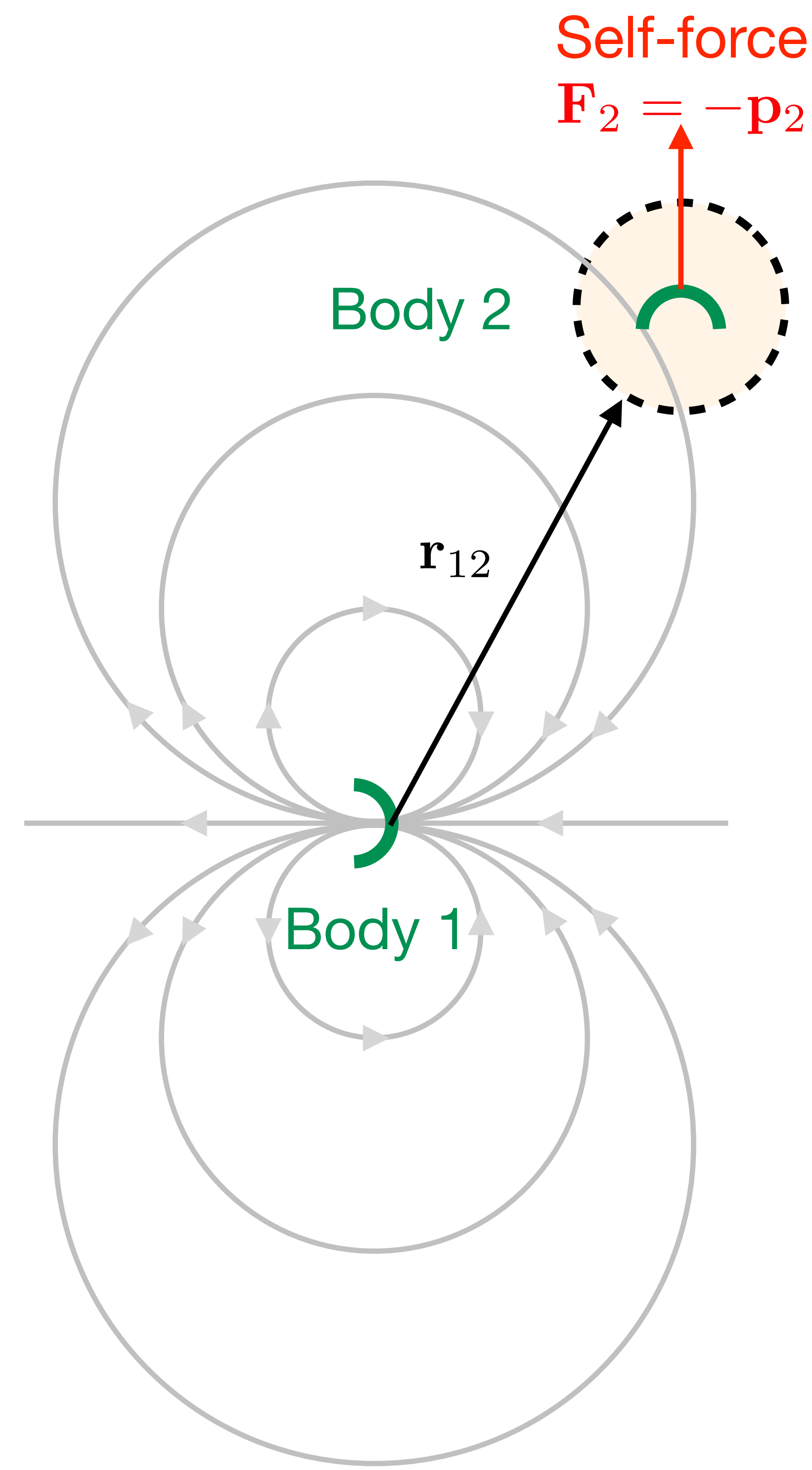
measured in the **absence** of body 2



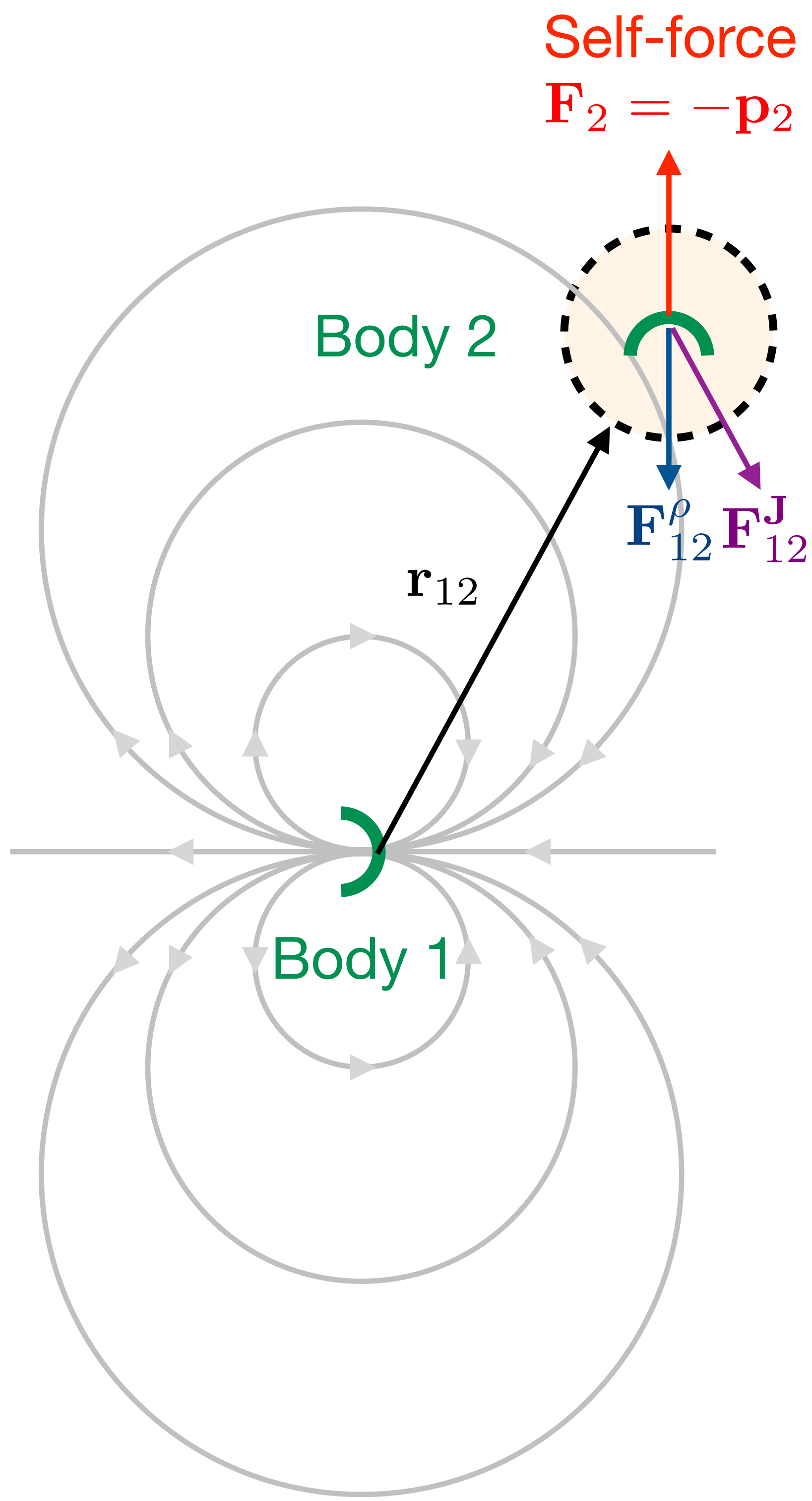
ADD second body

$$\mathbf{F}_2 = -\mathbf{p}_2$$

measured in the **absence** of body 1



each body modifies density and generates a current near other



Lead to:

1) Change in force on 2
due to change in density

$$\mathbf{F}_{12}^{\text{non-sym}} = -\frac{\beta_{\text{eff}}}{2\pi\rho_b}\frac{\mathbf{r}_{12}\cdot\mathbf{p}_1}{r_{12}^2}\mathbf{p}_2 + O(r_{12}^{-2})$$

2) Change in force on 2
due to flow field around

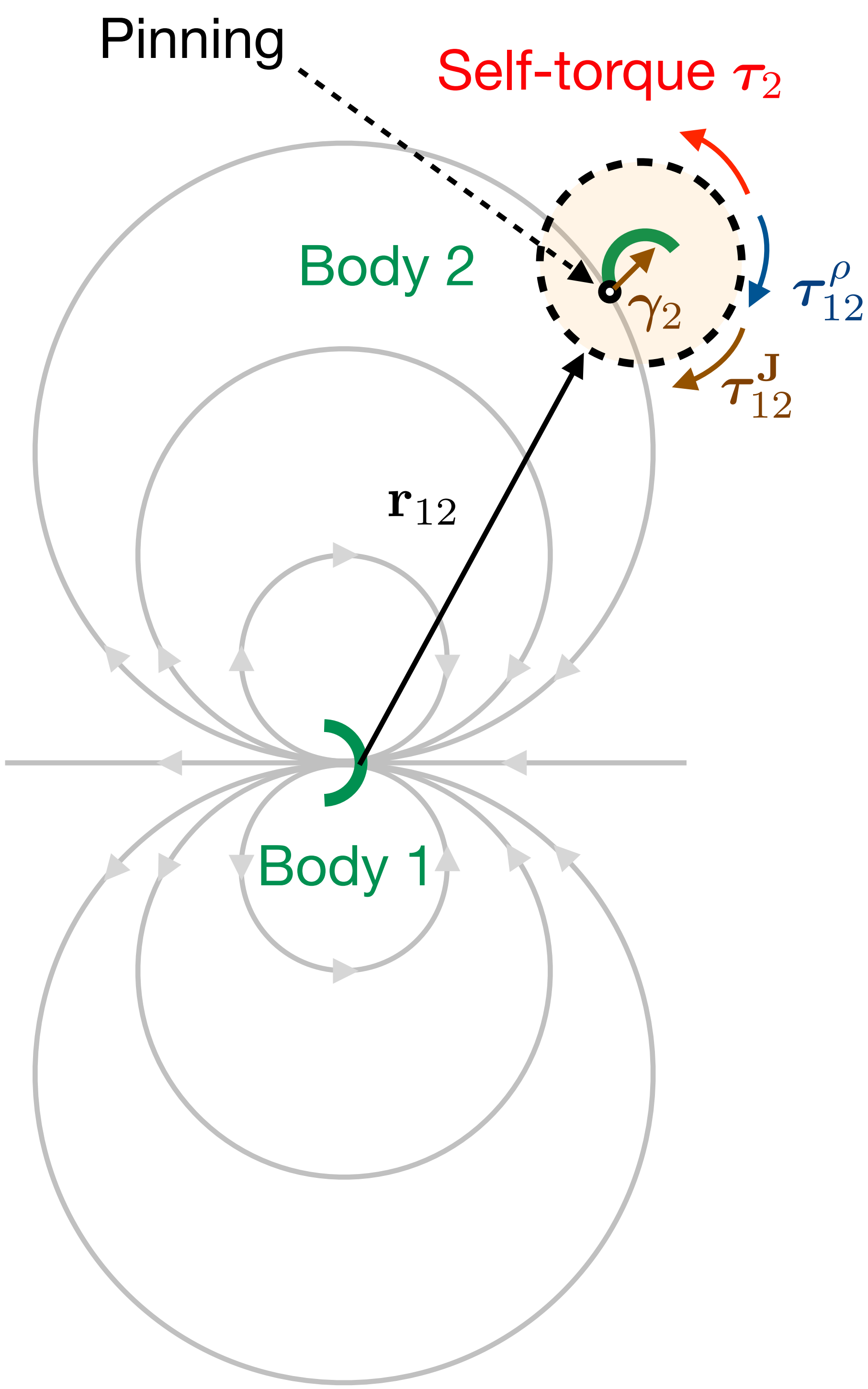
$$\mathbf{F}_{12}^{\text{sym}} = \frac{\mathbb{R}_2\mathbf{J}_1(\mathbf{r}_{12})}{\rho_b} + O(r_{12}^{-3})$$

$\mathbb{R}_2$ mobility matrix

$-\mathbf{J}_1/\rho_b$ drag velocity

$\beta_{\text{eff}} = \mu/D_{\text{eff}}$

Similarly can calculate torques due to interactions



density

$$\tau^a_{12} = \frac{\beta_{\rm eff}}{2\pi\rho_b}\frac{\mathbf{r}_{12}\cdot\mathbf{p}_1}{r^2_{12}}\tau_2 + O(r^{-2}_{12})$$

(leading-order if body 2 is chiral)

current

$$\tau^s_{12} = \frac{\gamma_2}{\rho_b} \times \mathbf{J}_1(\mathbf{r}_{12}) + O(r^{-3}_{12})$$

(leading-order if body 2 is not chiral)

γ_2 - response vector (2d) acts to align with current

1. **Long-range (power-law)**
2. **Anisotropic**
3. **No action reaction**

In sum:

$$\mathbf{F}_{12}^{\text{non-sym}} = -\frac{\beta_{\text{eff}}}{2\pi\rho_b}\frac{\mathbf{r}_{12}\cdot\mathbf{p}_1}{r_{12}^2}\,\mathbf{p}_2 + O(r_{12}^{-2})$$

(leading-order if body 2 is also dipole)

$$\mathbf{F}_{12}^{\text{sym}} = \frac{\mathbb{R}_2\mathbf{J}_1(\mathbf{r}_{12})}{\rho_b} + O(r_{12}^{-3})$$

(leading-order if body 2 is not a dipole)

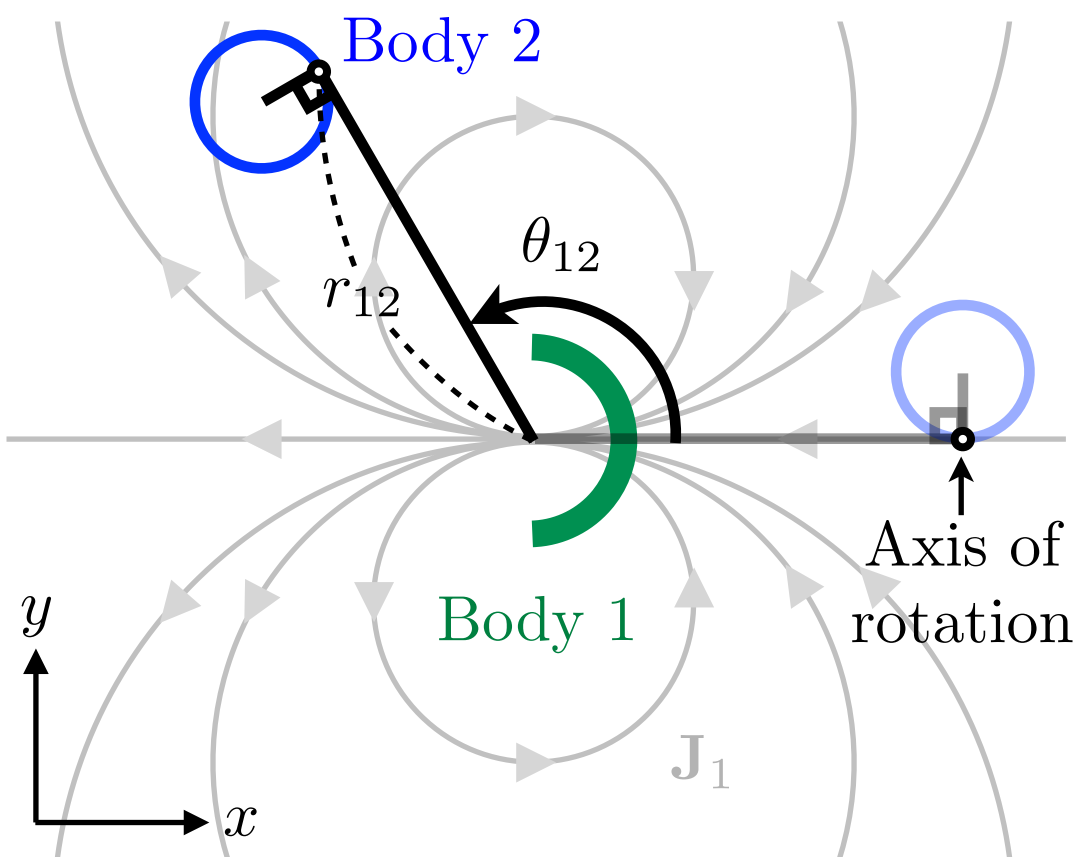
Expressed in terms of single body properties!

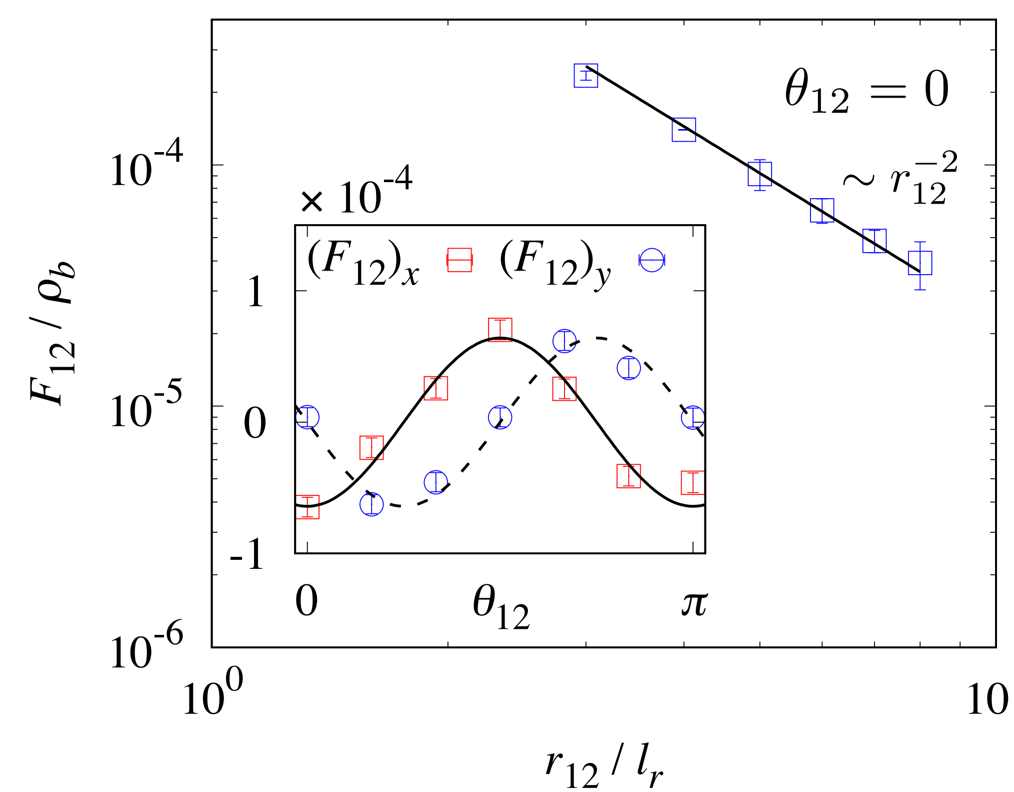
$$\mathbf{p}_i \quad \mathbb{R}_i$$

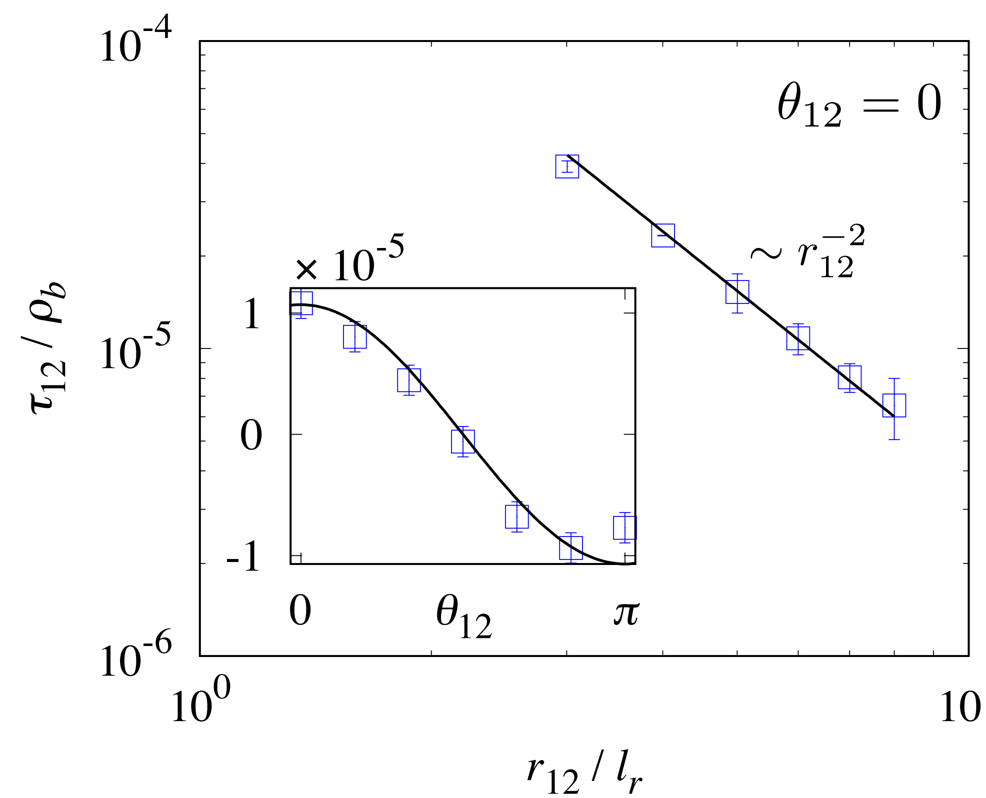
1. **Long-range (power-law)**
2. **Anisotropic**
3. **No action reaction**

Can have control by changing shape

Numerical verification - forces and torques (semi-circle + circle)





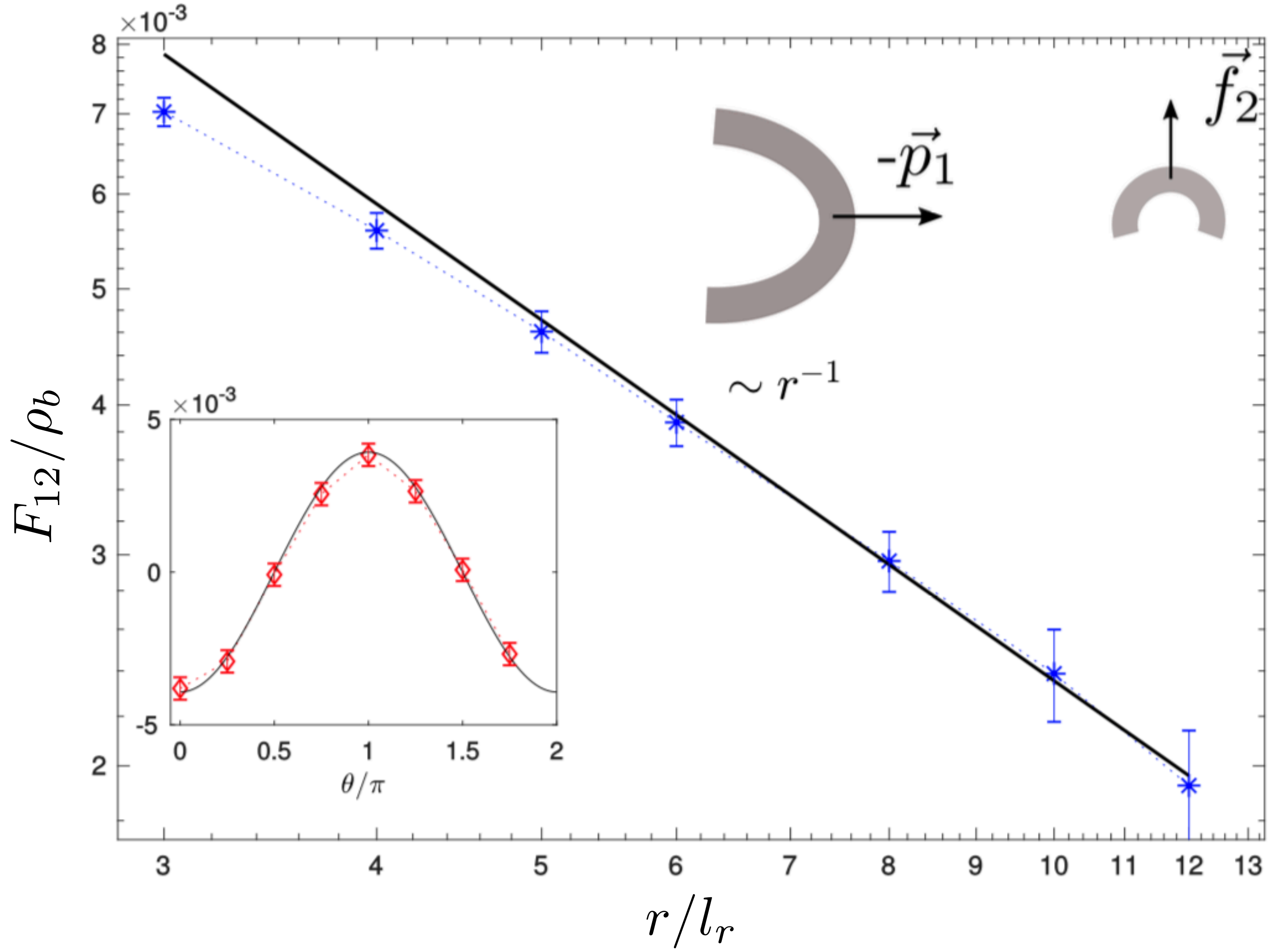


Since a disk has zero dipole moment,

$$\mathbf{F}_{12} \sim \mathbf{F}_{12}^{\mathbf{J}} \sim \frac{1}{r_{12}^2}$$

$$\boldsymbol{\tau}_{12} \sim \boldsymbol{\tau}_{12}^{\mathbf{J}} \sim \frac{1}{r_{12}^2}$$

Numerical verification - forces (semi-circle + semi-circle)



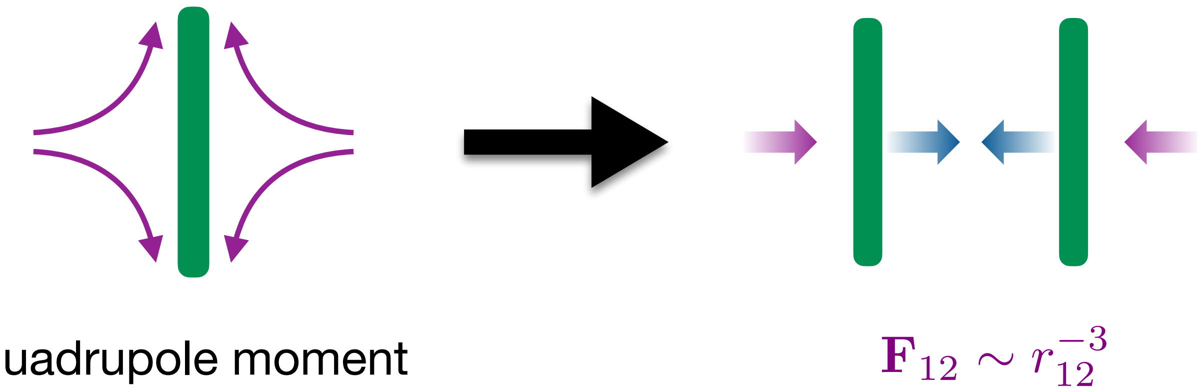
Since body 2 has nonzero dipole moment,

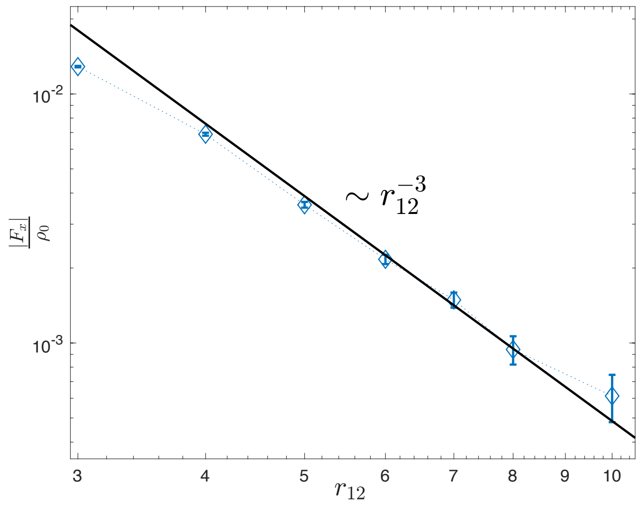
$$\mathbf{F}_{12} \sim \mathbf{F}^{\rho}_{12} \sim \frac{1}{r_{12}}$$

Comments:

- Easy to extend to higher dimension - r^{-d+1} and r^{-d}

- Can extend to higher multipoles, e.g., two rods





  - Previous studies on parallel rods/plates focused on mechanisms which give rise to exponential decay of the force with distance.
    Refs: [Ray, Reichhardt, Reichhardt, PRE (2014)], [Harder et al., J. Chem. Phys. (2014)] [Ni, Stuart, Bolhuis, PRL (2015)]
  - Our results show that the interaction is actually scale-free in the far field.

- Forces and torques calculated when objects static (adiabatic limit when moving)

Comments:

- With pairwise interactions results almost the same.
- Know nothing on response coefficients (can be negative)
- No action reaction - different from hydrodynamic interactions

$\mathbf{p}_1$ $\qquad\qquad$ $\mathbf{p}_2$

$$\mathbf{F}_{21} = 0 \qquad\qquad \mathbf{F}_{12} \neq 0$$

$$\mathbf{F}_{12}^{\text{non-sym}} = -\frac{\beta_{\text{eff}}}{2\pi\rho_b}\frac{\mathbf{r}_{12}\cdot\mathbf{p}_1}{r_{12}^2}\,\mathbf{p}_2 + O(r_{12}^{-2})$$

- Three (or more) objects far from each other forces additive
- In one-dimension (unpublished) can show forces not additive and do not decay with distance.

Summary and Outlook

- Single asymmetric object in active bath experiences forces.
- Generated long-range (power-law) modulations of density and a current
- Lead to long-range interactions between two bodies
- In far-field can express in terms of single-body properties
- Can show without interactions and with pairwise interactions

- Handles not turned:
  torques on active particles, non-pairwise interactions between active particles
- Response matrices - not much known.
  Near-field interactions? (some explored before)
- *Can control the interactions by the shape*
  - self-assembly?
  (maybe combine with ideas from
  R Soto, R Golestanian, PRL 112, 068301 (14))