Wormholes and Entanglement

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Our papers with Curt

Brane death and dynamics from the Born-Infeld action
Curtis G. Callan (Princeton U.), Juan Maldacena (Rutgers U., Piscataway)
e-Print: **hep-th/9708147**

D-brane approach to black hole quantum mechanics
Curtis G. Callan, Juan Maldacena
e-Print: **hep-th/9602043**

Extremal black holes as fundamental strings
Curtis G. Callan, Juan Maldacena, Amanda W. Peet
e-Print: **hep-th/95101**

Magnetic fields and fractional statistics in boundary conformal field theory
Curtis G. Callan, Igor R. Klebanov, Juan Maldacena, A. Yegulalp
e-Print: **hep-th/9503014**

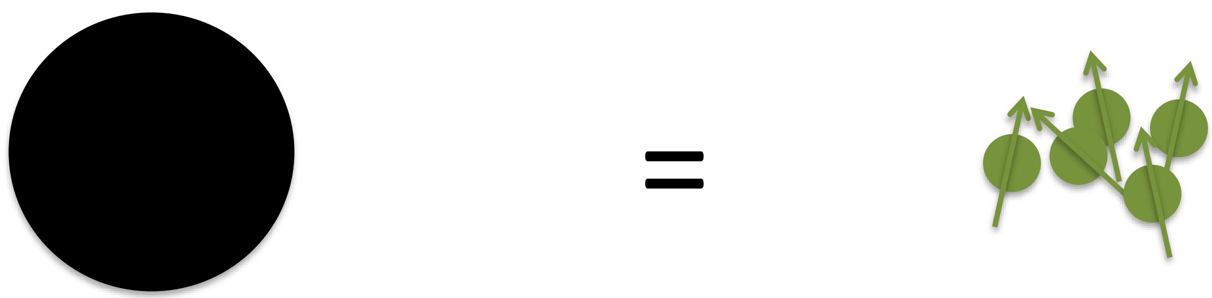
Exact solution of a boundary conformal field theory
Curtis G. Callan, Igor R. Klebanov, Andreas W.W. Ludwig, Juan Maldacena
e-Print: **hep-th/9402113**

Outline

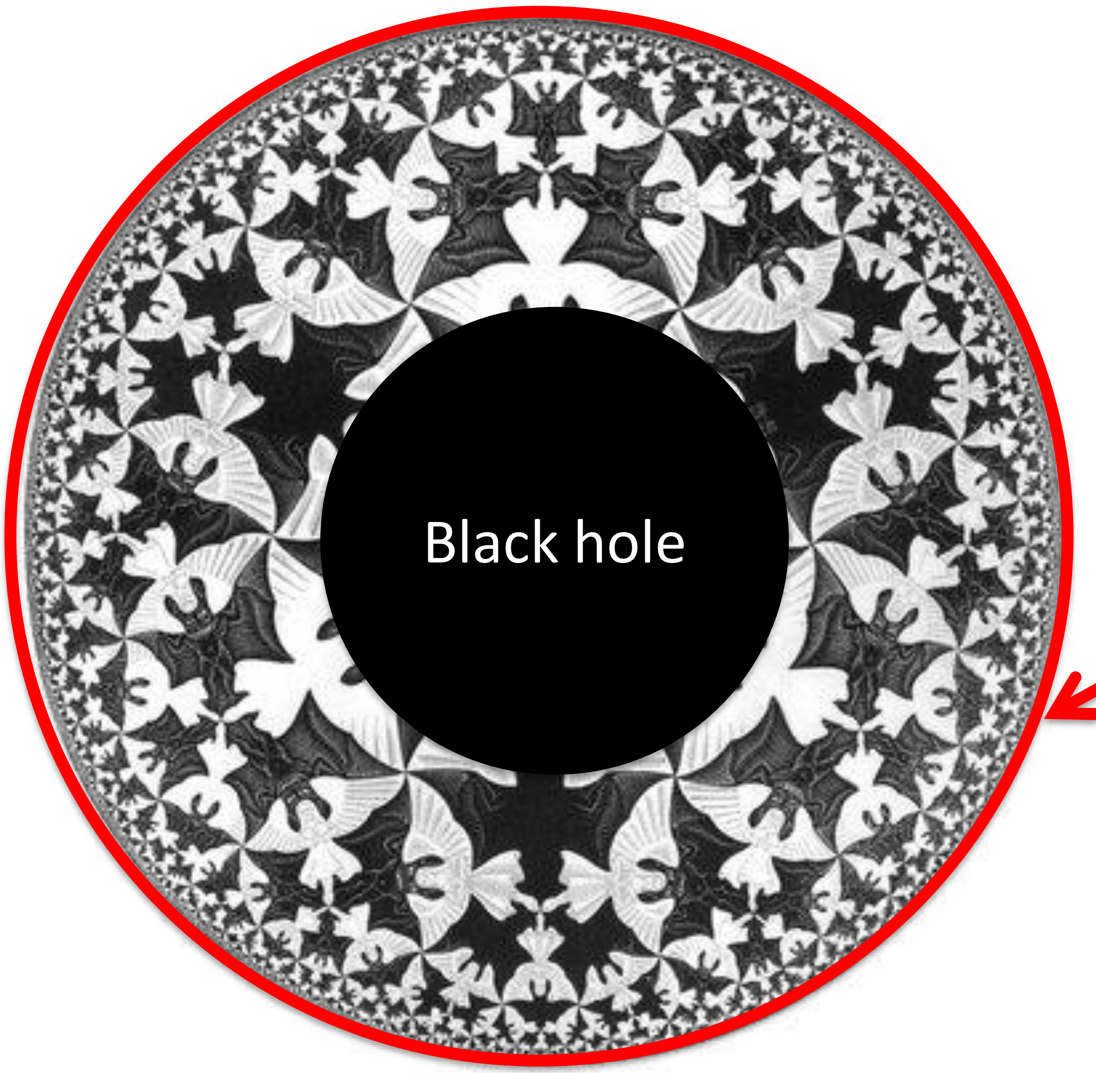
- Black holes as quantum systems
- Schwarzschild wormhole and entangled states.
- Traversable wormholes.
- Traversable wormhole solution in 4d.

Black holes as quantum systems

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits).



One reason is AdS/CFT…



Hot fluid made out of
<u>very strongly</u>
Interacting particles.

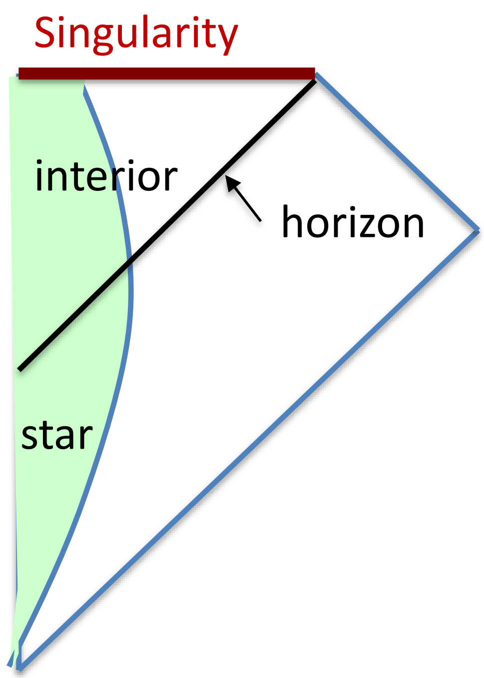
More recent toy models

- SYK model = N interacting Majorana fermions

Sachdev-Ye-Kitaev

- → many properties in common with nearly extremal charged black holes.

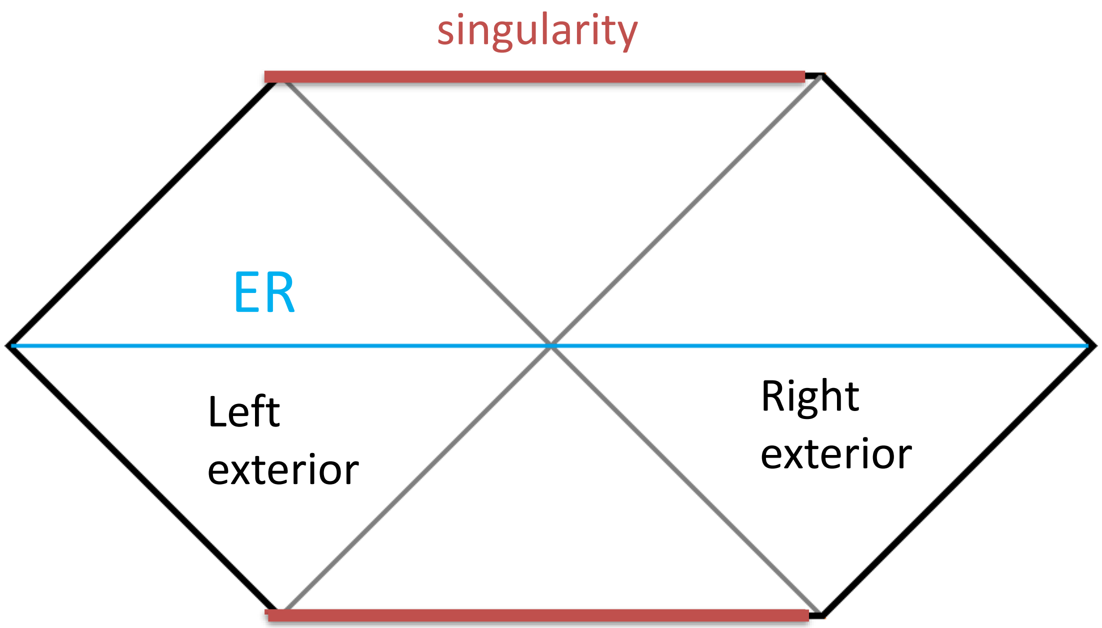
Geometry of a Black Hole made from collapse



Oppenheimer Snyder 1939

One exterior, one interior.

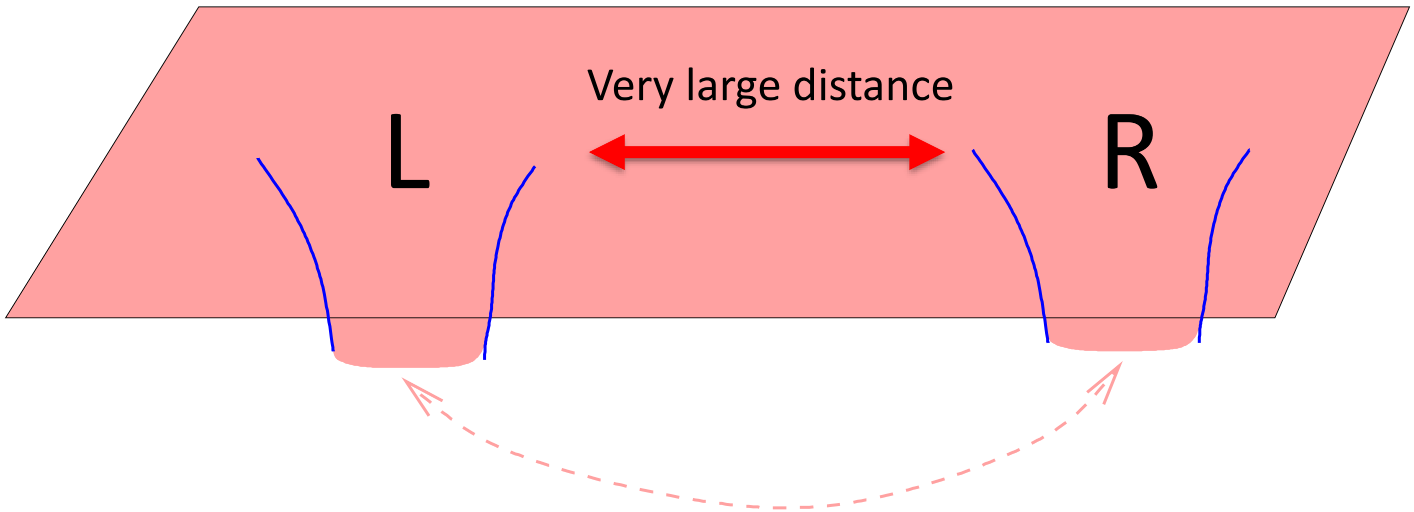
Full Schwarzschild solution

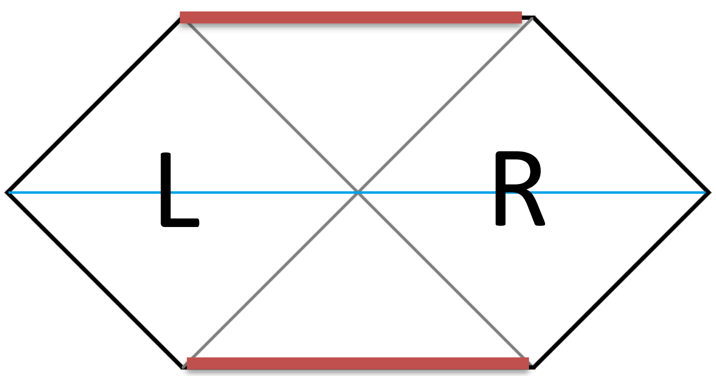


Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

Vacuum solution.
Two exteriors, sharing the interior.

Wormhole interpretation.





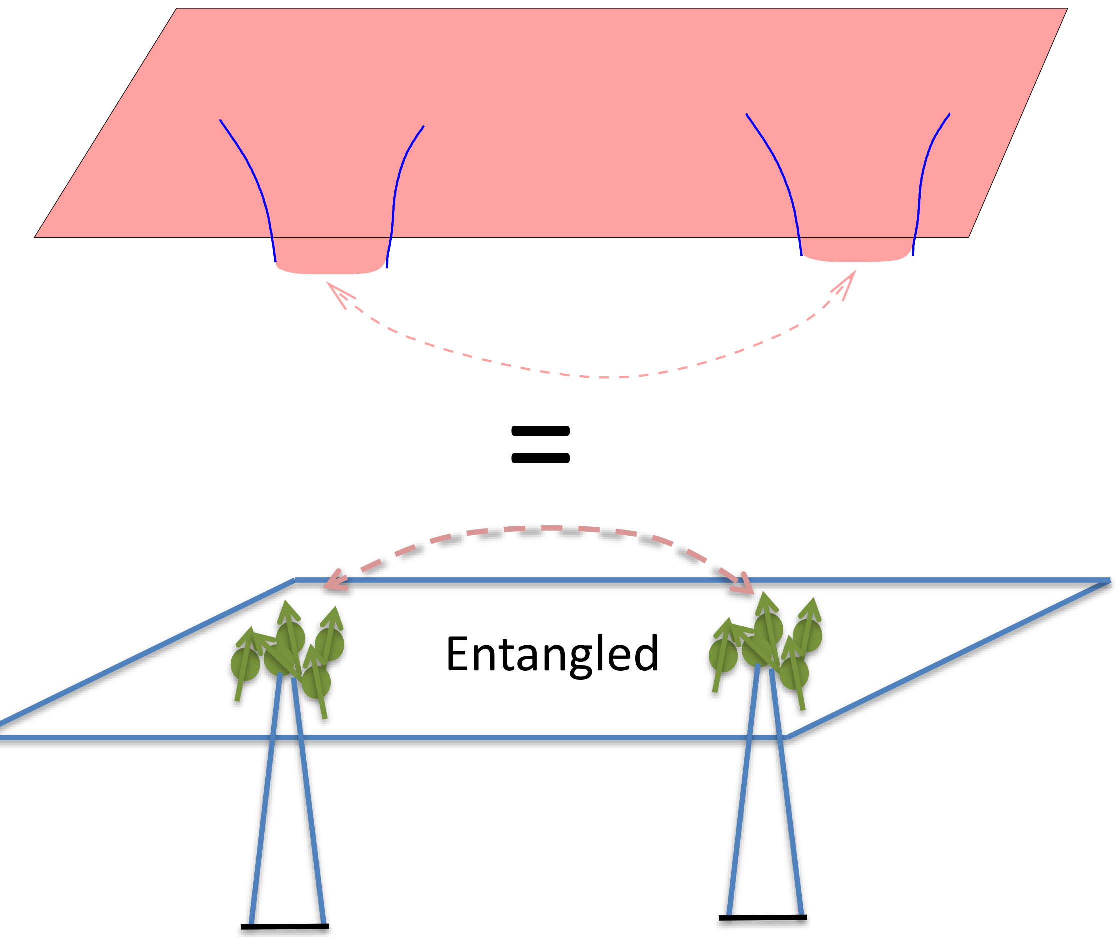
Non traversable

No signals

No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

Wormhole and entangled states



Connected through the interior

W. Israel
J.M.

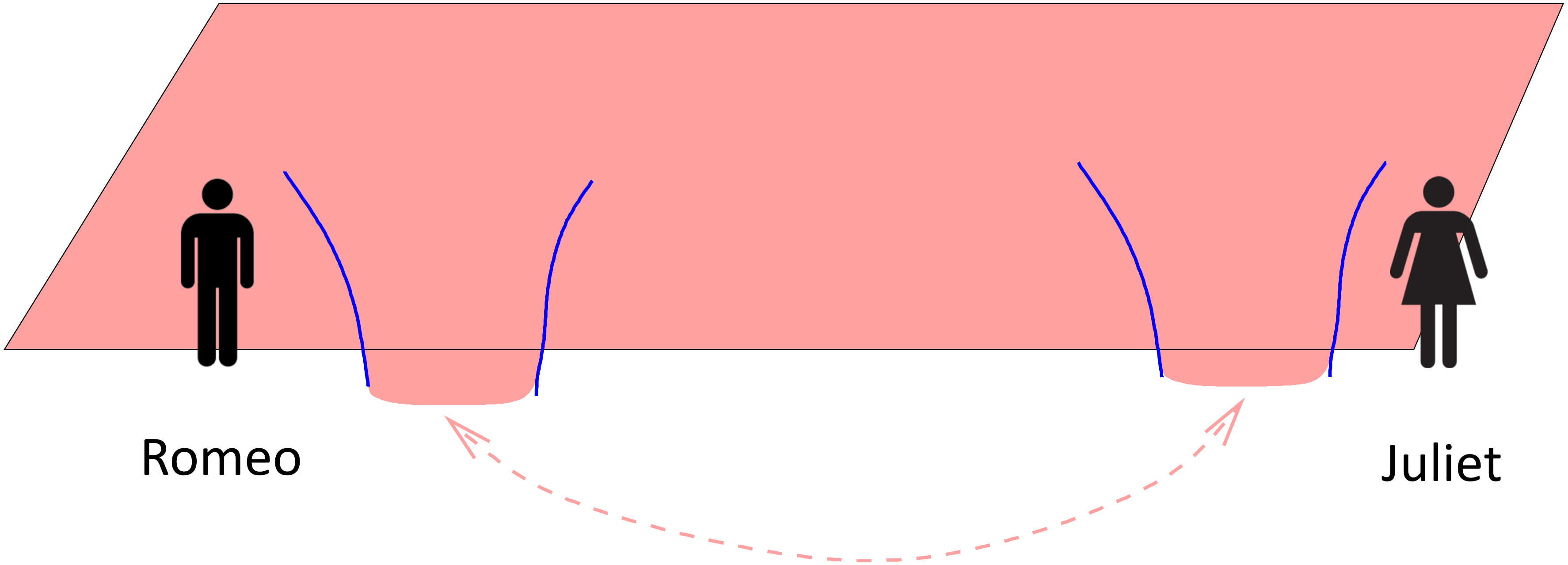
In a particular entangled state

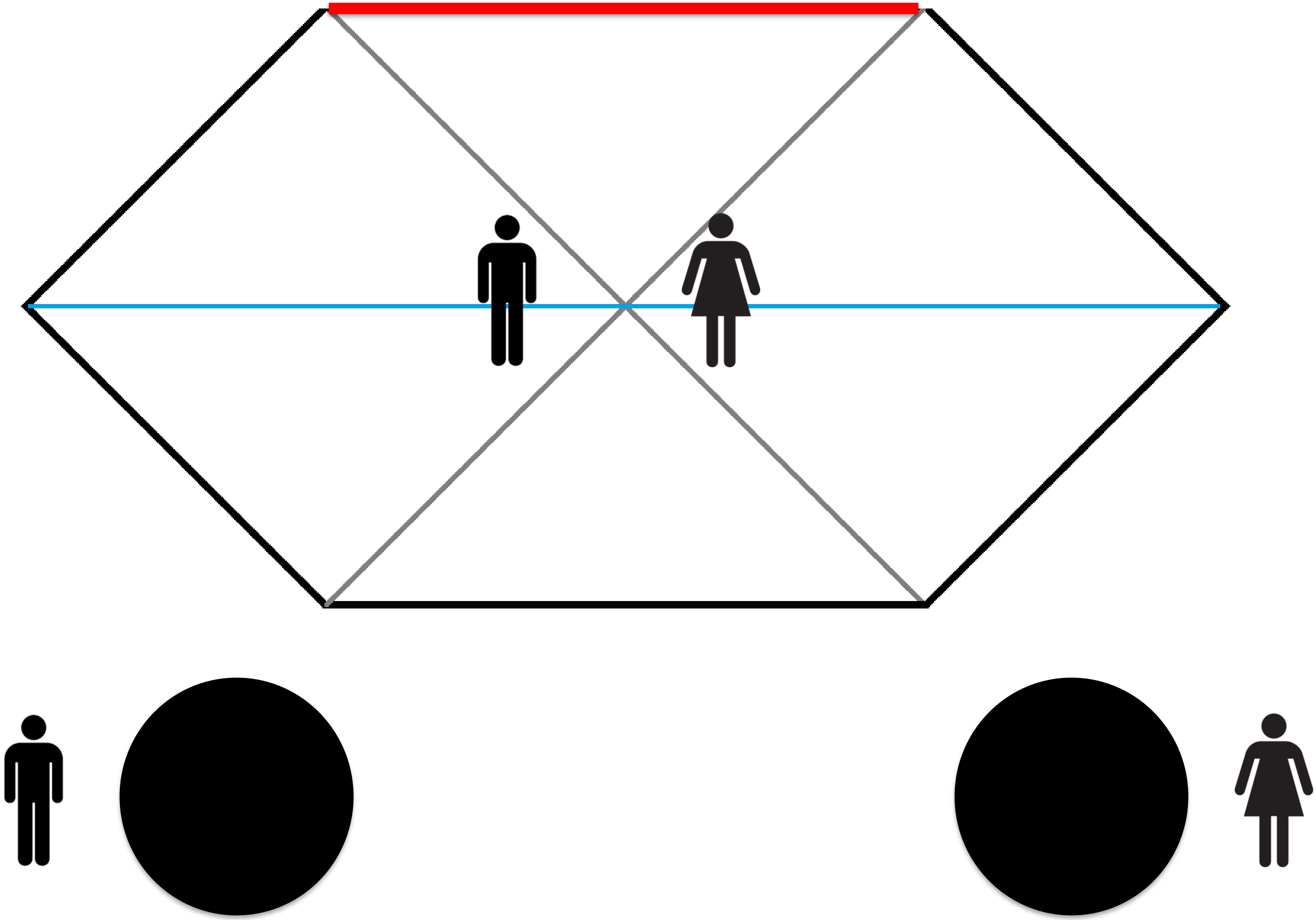
$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

ER = EPR

J.M. , Susskind

A forbidden meeting





Is there a better way ?

- Traversable wormholes
- Quantum teleportation

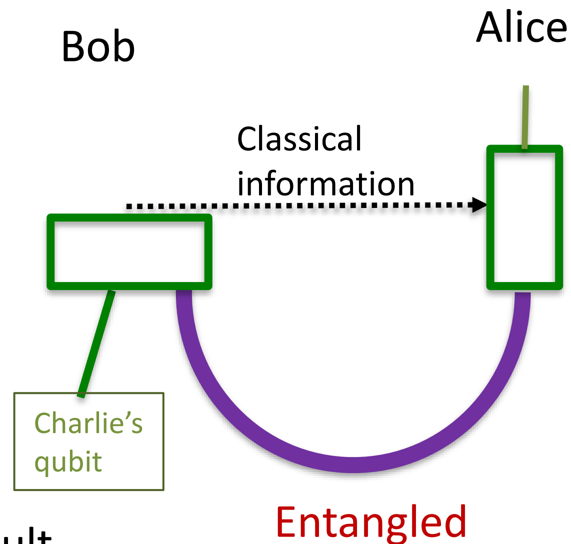
Gao, Jafferis, Wall, …

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

Quantum teleportation

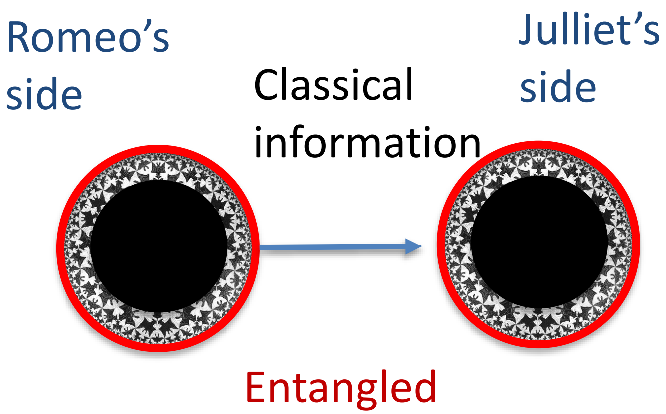
- Bob and Alice share an entangled pair of qubits.
- Charlie gives Bob a qubit and he wants to send it to Alice.
- Bob does a joint measurement of Charlie's qubit and his qubit.
- Sends the the result to Alice as classical information
- Alice does an operation on his qubit that depends on Bob's result.
- Alice gets the qubit.

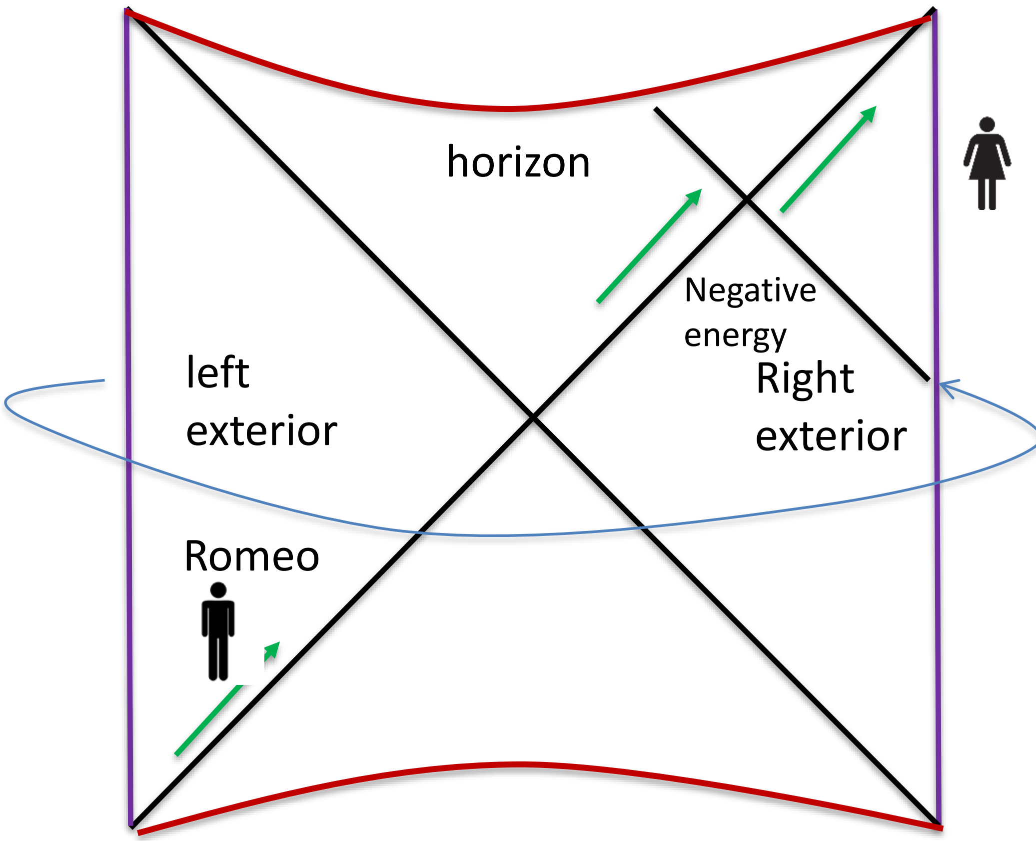


- Resources needed to send a qubit: One entangled qubit and 2 bits of classical information.

Would you like to be teleported ?

Teleportation through the wormhole





Gao, Jafferis, Wall

- What did Romeo feel ?
- Nothing if it is spherically symmetric.

Conclusions

- Entangled black holes can display a geometric connection.

- Entanglement is related to wormholes. ER=EPR.

- Teleportation is related to travel through wormholes.

Related reasoning inspired the following solution

Wormholes in 4 dimensions,
in the Standard Model + gravity

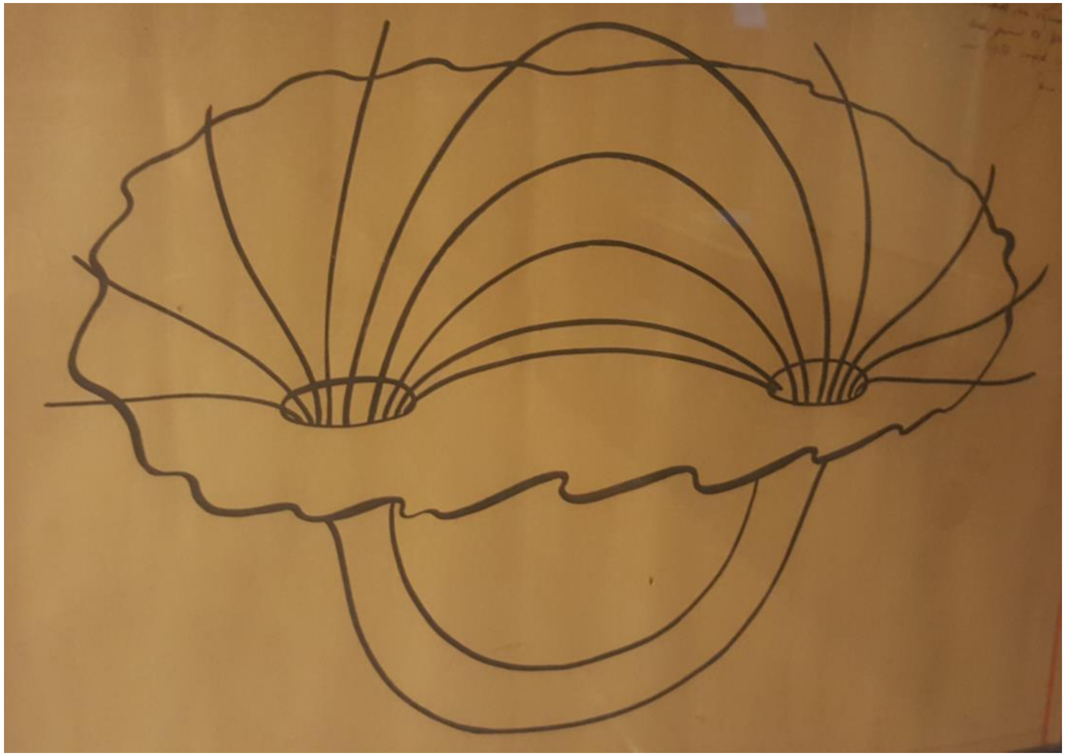
Based on work with:

Alexey Milekhin

Fedor Popov

Related to previous work with Xiaoliang Qi

Drawing by John Wheeler, 1966



Charge without charge.
Mass without mass

Spatial geometry. Traversable wormhole

Recall classic results

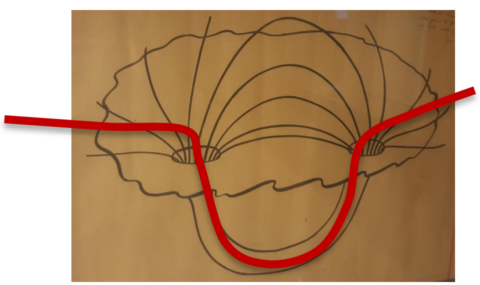
There are no science fiction wormholes!

- No wormhole allows you to travel faster than the speed of light in the ambient space.
- Forbidden by:
- I) Einstein equations.
- II) The Achronal Average Null Energy Condition

Friedman Schleich, Witt, Galloway, Woolgar
Gao Wald

Not yet proven in a general spacetime, but believed to hold in QFT

$$\int dx^{-} T_{--} \geq 0$$



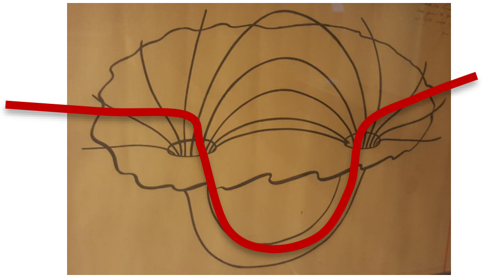
Achronal = points along null line are not timelike separated = fastest line

Longer wormholes?

- What if it takes longer to go through the wormhole ?
- Not possible in classical physics due to the Null Energy Condition.

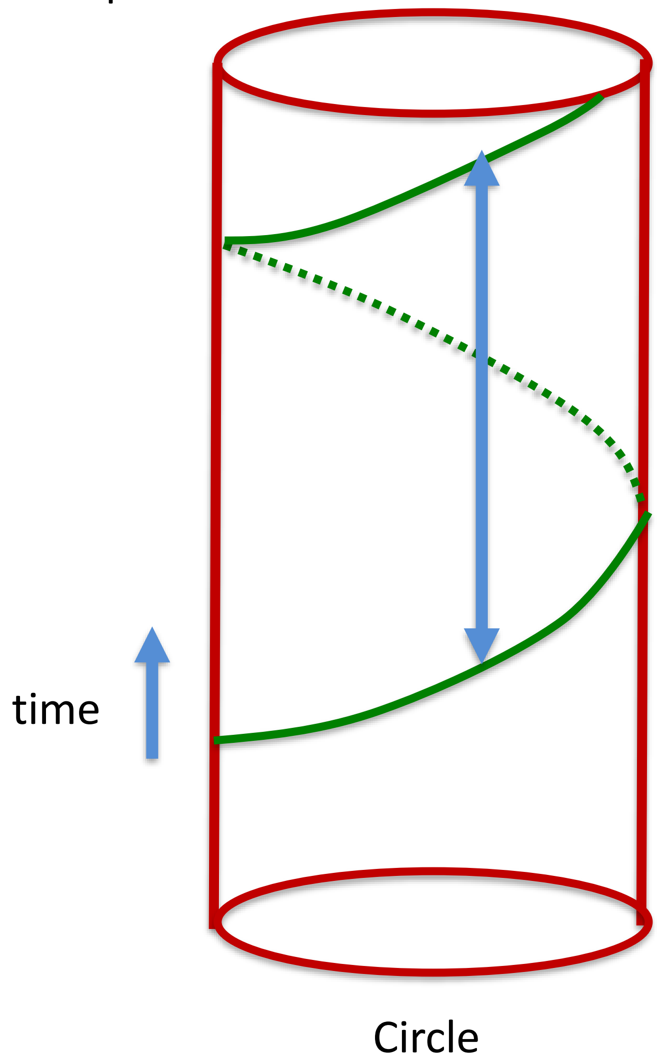
Topological censorship: Friedman Schleich, Witt, Galloway, Woolgar

- Possible in quantum physics.
- → We need quantum effects to find a solution. Casimir-like energy.



Negative null energy in QFT

Eg. Two spacetime dimensions



$$T_{--} < 0$$

$$E \propto -\frac{c}{L}$$

Negative Casimir energy

Quantum effect

The null energy condition does not hold for null lines that are not achronal!

Some necessary elements

- We need something looking like a circle to have negative Casimir energy.
- Large number of bulk fields to enhance the size of quantum effects.
- We will show how to assemble these elements in a few steps.

The theory

$$S = \int d^4x \left[R - F^2 + i\bar{\psi} \not{D} \psi \right]$$

Einstein + U(1) gauge field + massless charged fermion

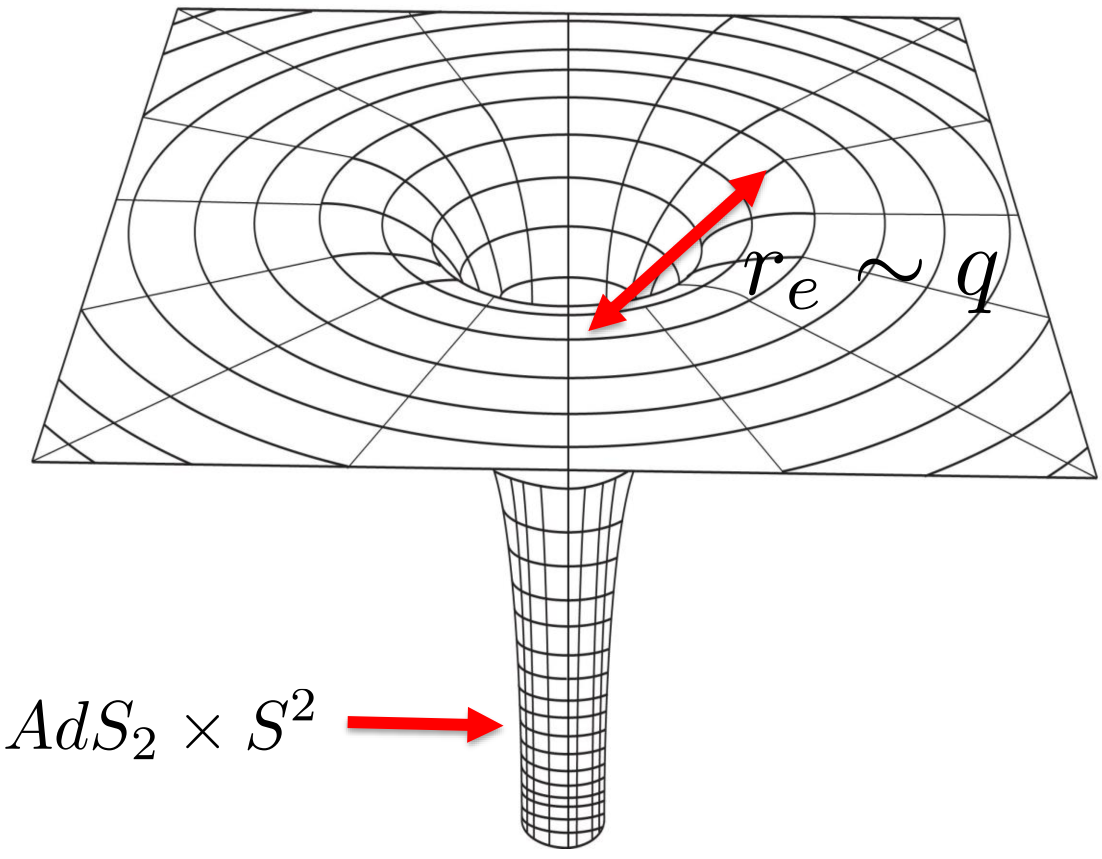
Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

$l_{\text{Planck}} = 1$

The first solution: Extremal black hole

Magnetic charge q

$$\int_{S^2} F = q = \text{integer}$$



$$M = q$$

$$l_{\text{Planck}} = 1$$

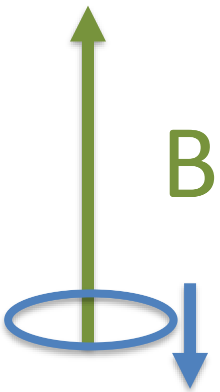
Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.

Crucial for the Callan-Rubakov effect…

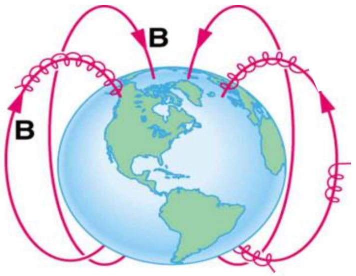
- Explained by an anomaly argument

Ambjorn, Olesen

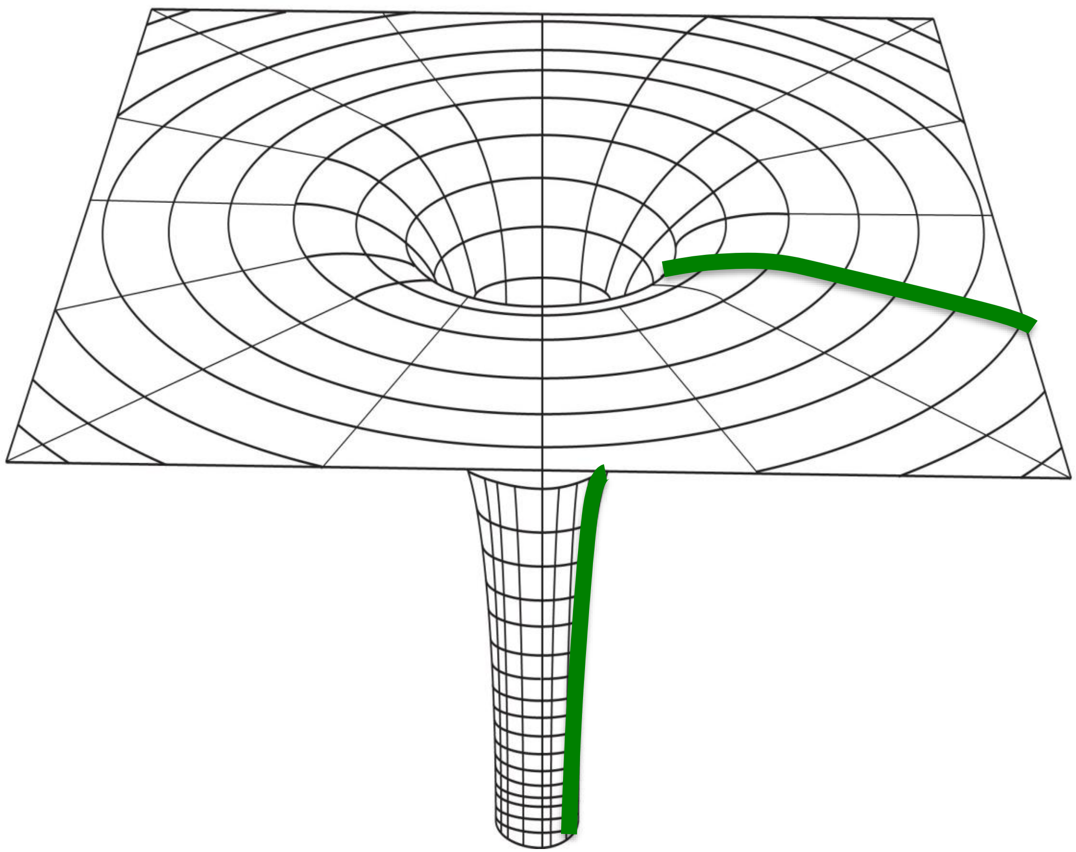


Motion of charged fermions

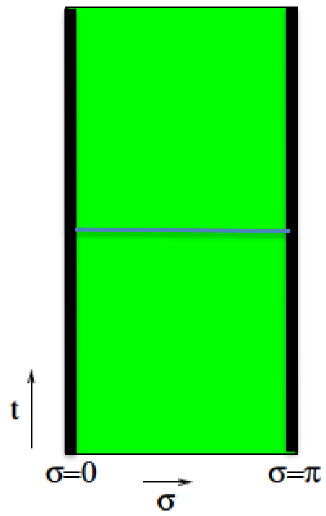
- Degeneracy = q = flux of the magnetic field on the sphere. Form a spin j, representation of SU(2), 2j +1 =q.
- We effectively get q massless two dimensional fermions along the time and radial direction.
- We can think of each of them as following a magnetic field line.

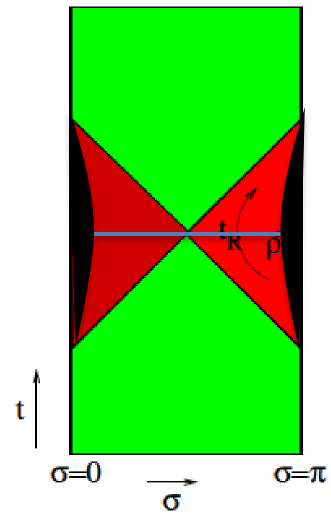


q massless two dimensional fields, along field lines.



Nearly AdS$_2$





$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

Global

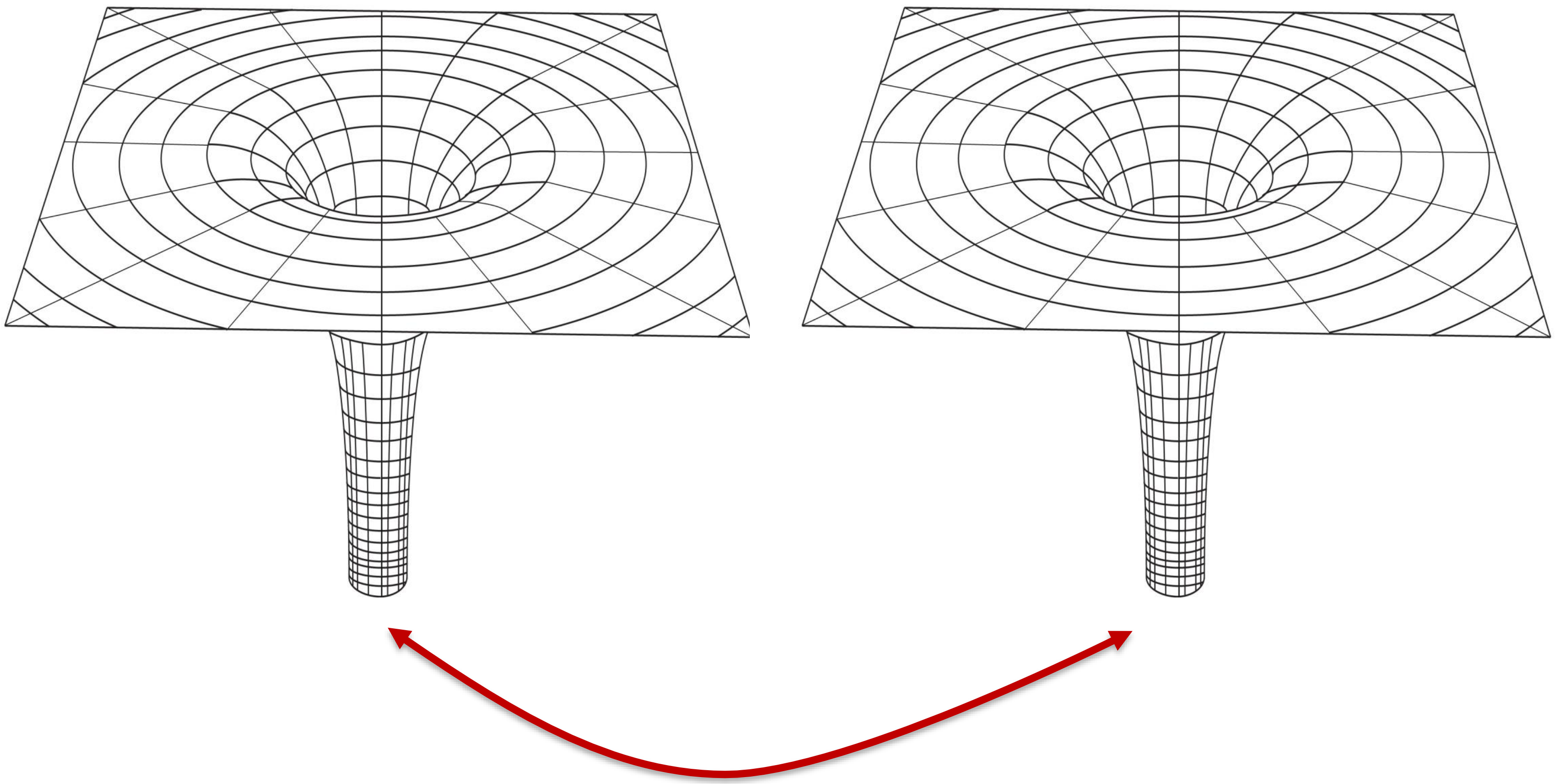
$$ds^2 = -(r^2 - 1)d\tau^2 + \frac{dr^2}{(r^2 - 1)}$$

Thermal/Rindler

Connect them to flat space, so that t is an isometry.
The acquire non-zero energy when the throat has finite length

$$M - q = q^3 T^2 = \frac{q^3}{\beta^2}$$

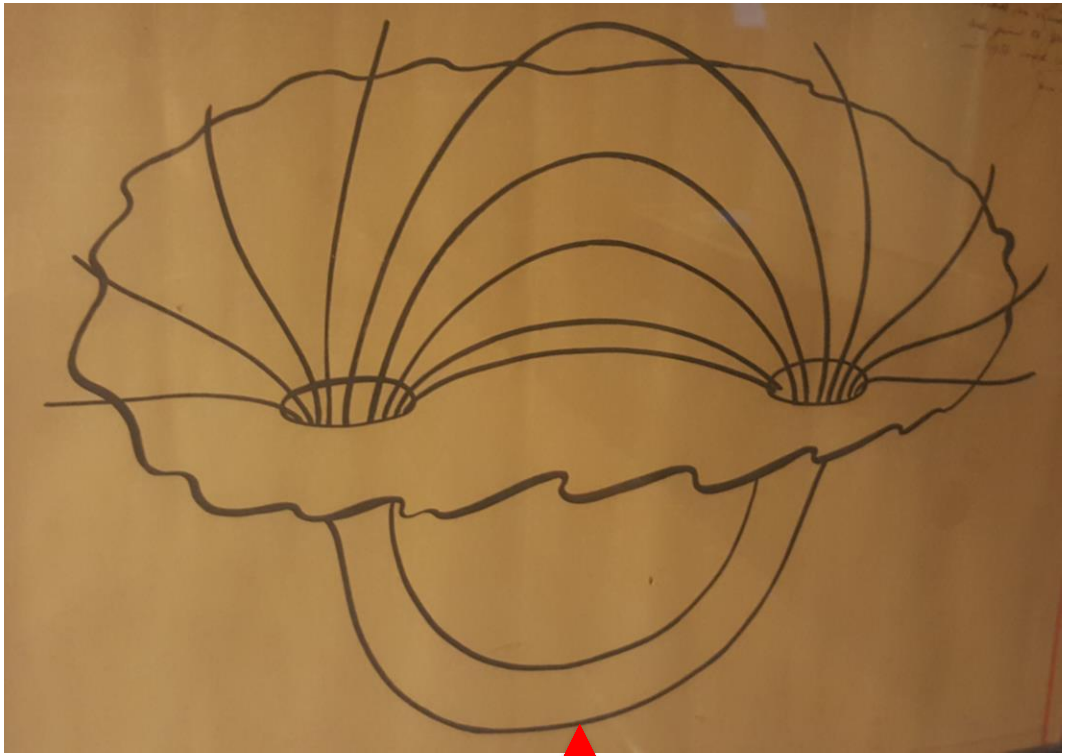
Connect a pair black holes



connect and in global AdS_2

Connect a pair black holes

Positive magnetic charge



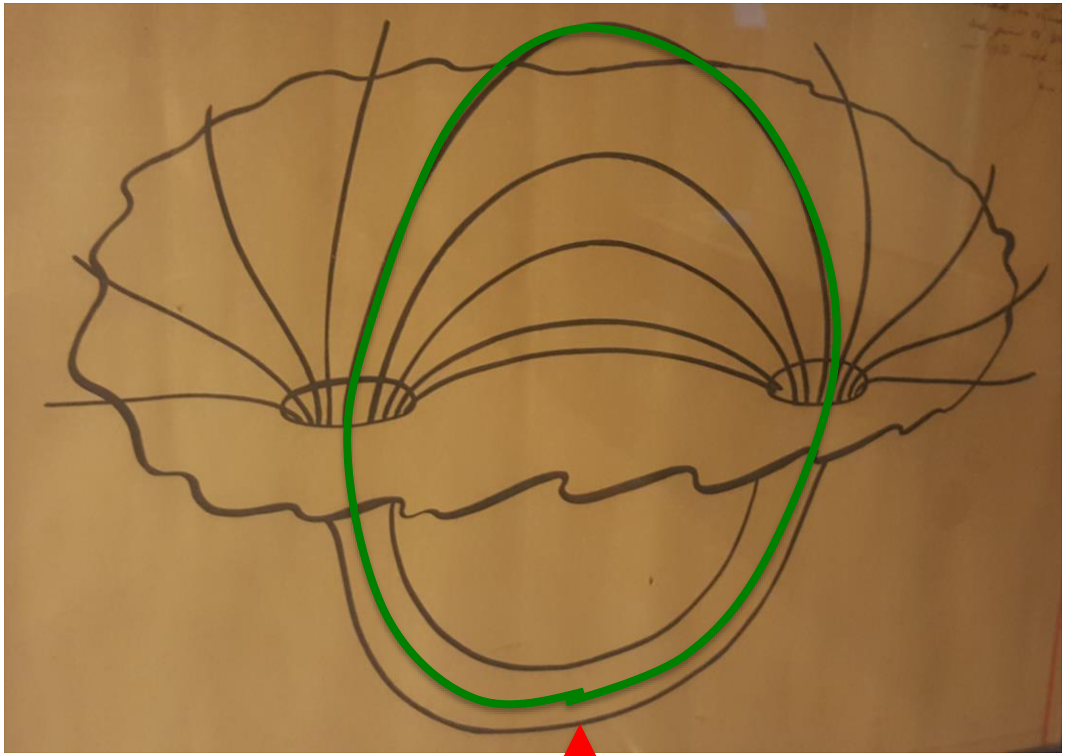
Negative magnetic charge

Not a solution yet.
Not a black hole.

Nearly $AdS_2 \times S^2$ wormhole of finite length

Fermion trajectories

Positive magnetic charge



Negative magnetic charge

Charged fermion moves along this closed circle.

Casimir energy

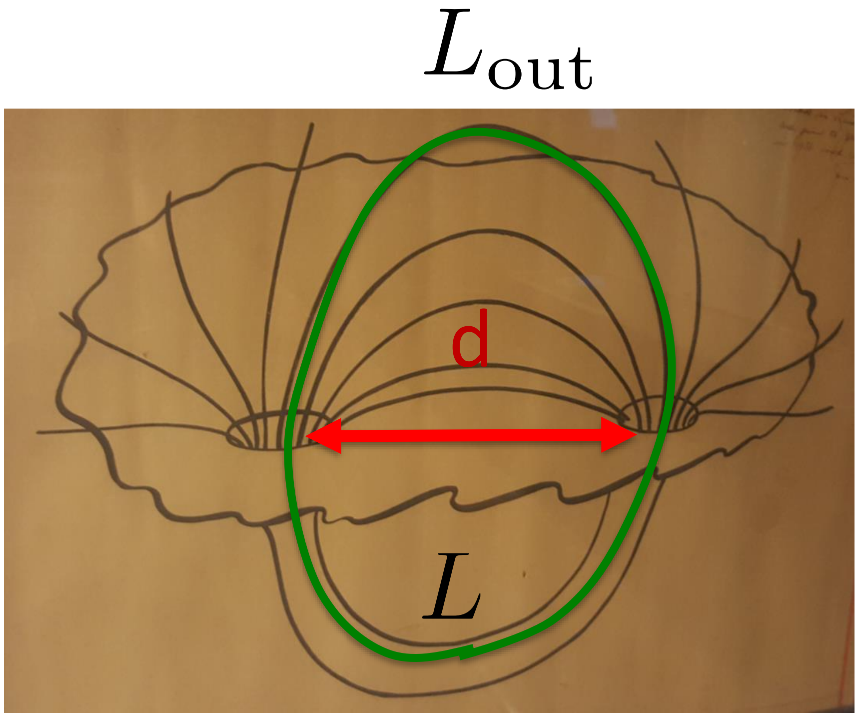
Assume: "Length of the throat" is larger than the distance.

$L \gg L_{out} > d$

Casimir energy is of the order of

$$E \propto -\frac{q}{L}$$

Full energy also need to take into
Account the conformal anomaly because
AdS_2 has a warp factor.
That just changes the numerical factor.



Finding the solution

Solve Einstein equations in the throat region with the negative quantum stress tensor

=

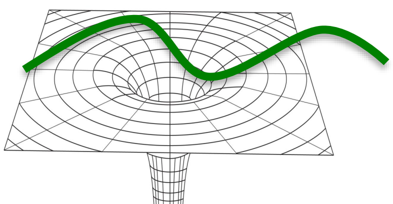
Balance the classical curvature + gauge field energy vs the Casimir energy.

$$M - q = \frac{q^3}{L^2} - \frac{q}{L}, \qquad \frac{\partial M}{\partial L} = 0 \longrightarrow L \sim q^2$$

Now the throat is stabilized. Negative binding energy.

$$E_{\text{binding}} = M - q = -\frac{1}{q} = -\frac{1}{r_s}$$

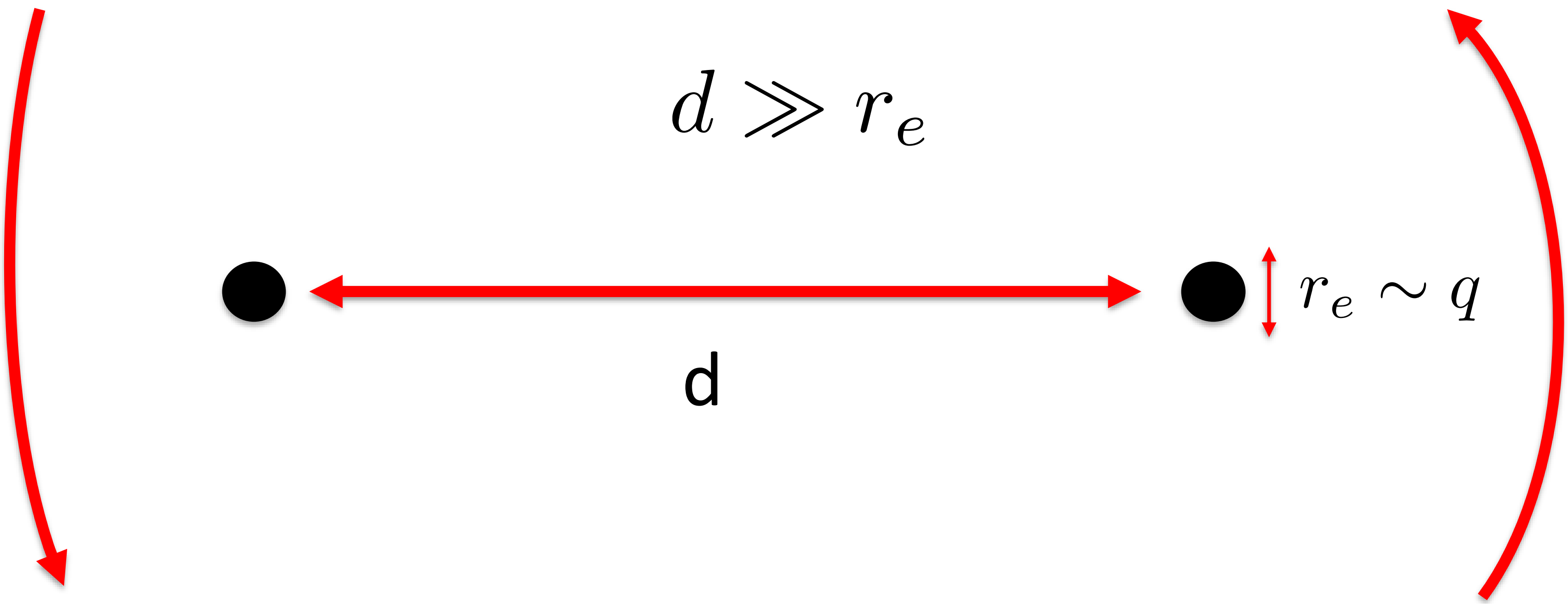
Very small. Only low energy waves can explore it



This is not yet a solution in the outside region:

The two objects attract and would fall on to each other

Adding rotation



$$\Omega = \sqrt{\frac{r_e}{d^3}}$$ Kepler rotation frequency

Some necessary inequalities

$$L \sim q^2$$

From stabilized throat solution

$$d \ll L \longrightarrow d \ll q^2$$

Black holes close enough to that Casmir energy computation was correct.

$$\sqrt{\frac{q}{d^3}} = \Omega \ll \frac{1}{L} \longrightarrow q^{\frac{5}{3}} \ll d$$

Black holes far enough so that they rotate slowly compared to the energy gap.

Kepler
rotation frequency

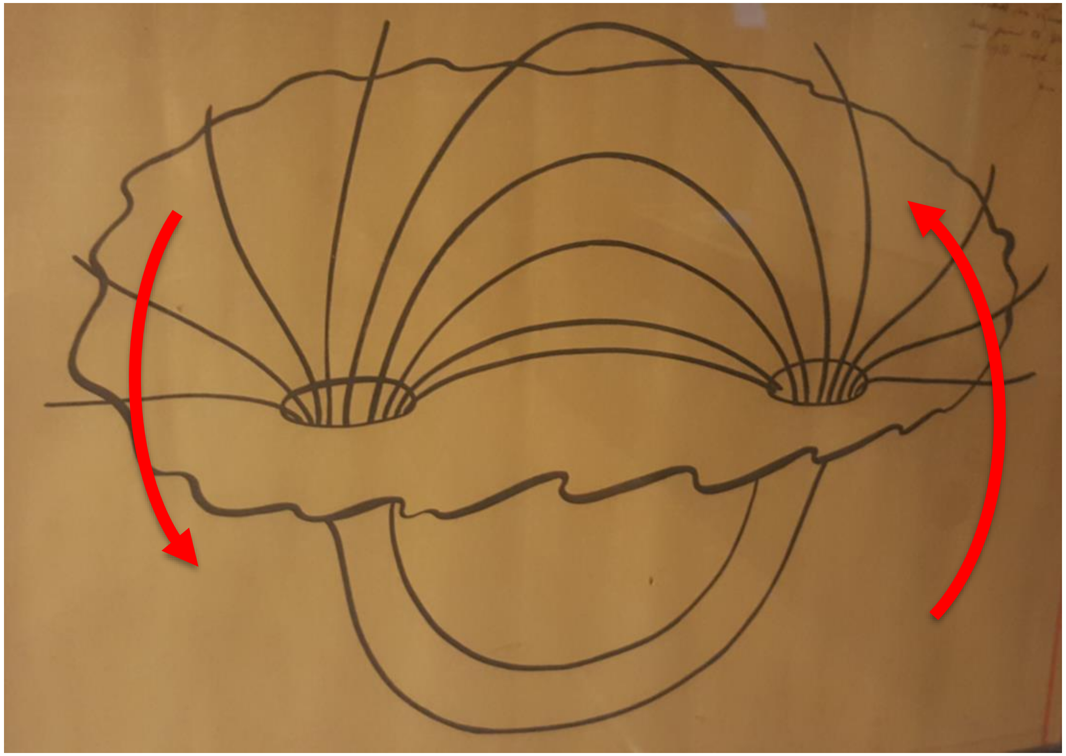
Unruh-like temperature less than energy gap

They are compatible

$$q^{\frac{5}{3}} \ll d \ll q^2$$

Other effects we could think off are also small :
can allow small eccentricity, add electromagnetic and gravitational radiation, etc. Has a finite lifetime.

Final solution



Looks like the exterior of two near extremal black holes. But they connected.
But there is no horizon!. Zero entropy solution.
It has a small binding energy.

Wormholes in the Standard Model

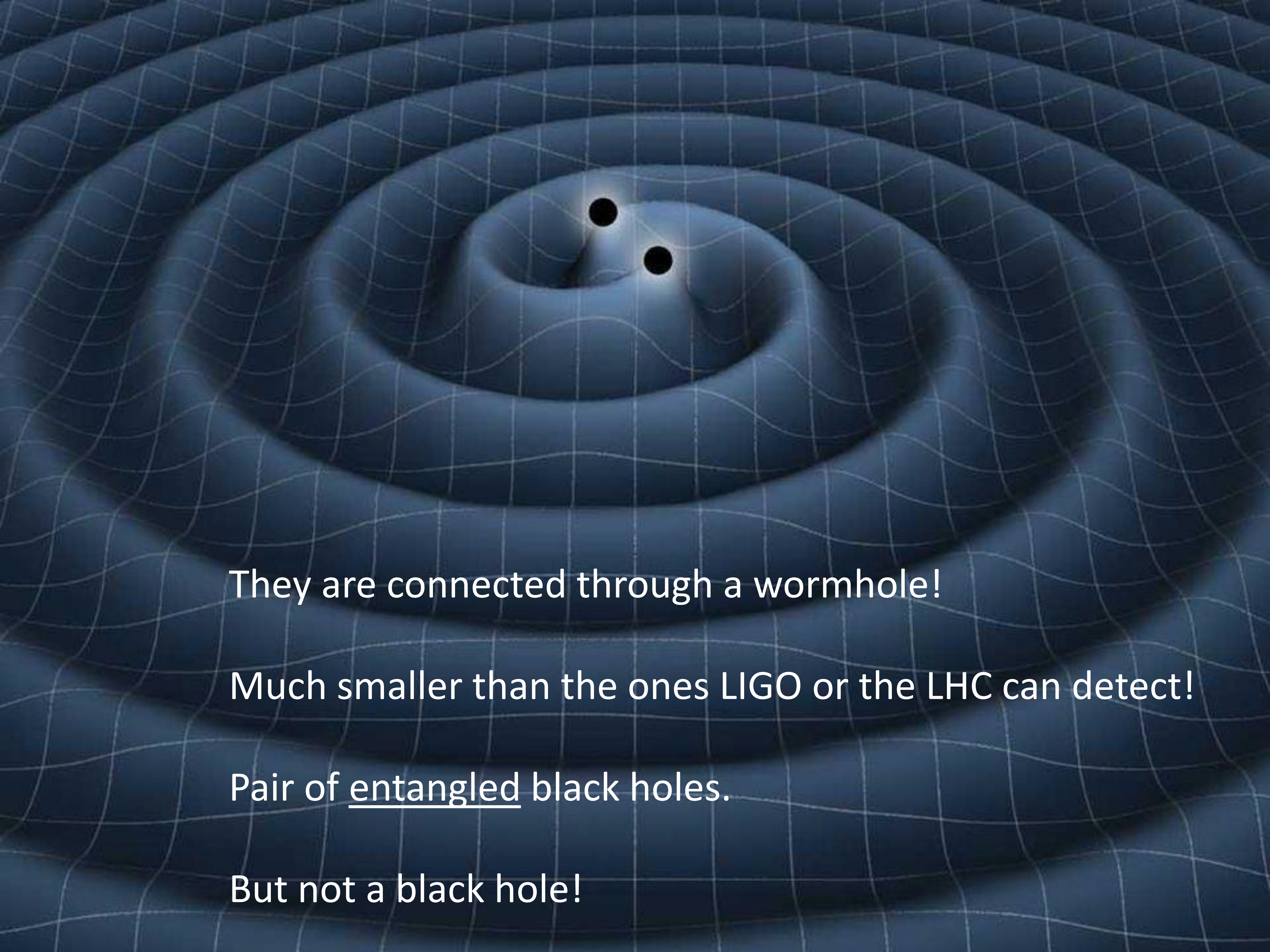
If nature is described by the Standard Model at short distances and d is smaller than the electroweak scale,

$$1 \ll q \ll 10^8$$

Distance d smaller than electroweak scale.

If the standard model is not valid → similar ingredients might be present in the true theory.

That it can exist, does not mean that it is easily produced by some natural or artificial process.


They are connected through a wormhole!
Much smaller than the ones LIGO or the LHC can detect!
Pair of entangled black holes.
But not a black hole!

Entropy and entanglement

- Total spacetime has no entropy and no horizon.
- If we only look at one object → entanglement entropy = extremal black hole entropy
- Wormhole = two entangled black holes
- Total Hamiltonian $H = H_L + H_R + H_{\rm int}$

Generated by fermions in exterior

Conclusions

- We displayed a solution of an Einstein Maxwell theory with charged fermions.
- It is a traversable wormhole in four dimensions and with no exotic matter.
- It balances classical and quantum effects.
- It has a non-trivial spatial topology.
- It has no horizon and no entropy.
- Can be viewed as a pair of entangled black holes.