Nature's Simplest Amorphous Solids

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Special thanks to many, including…
Horacio Castillo (Ohio U), Weiqun Peng (GWU),
Xiaoming Mao (U Michigan), Bing Lu (Nanyang U),
Xiangjun Xing (SJTU), Annette Zippelius (U Göttingen)
and of course Nigel Goldenfeld (U Illinois)

WHAT'S THE GOAL?

- Minimal field theory for vulcanized media (VM)
- Origins
- Implications
- Why VM furnish nature's simplest amorphous solids
- Open challenges

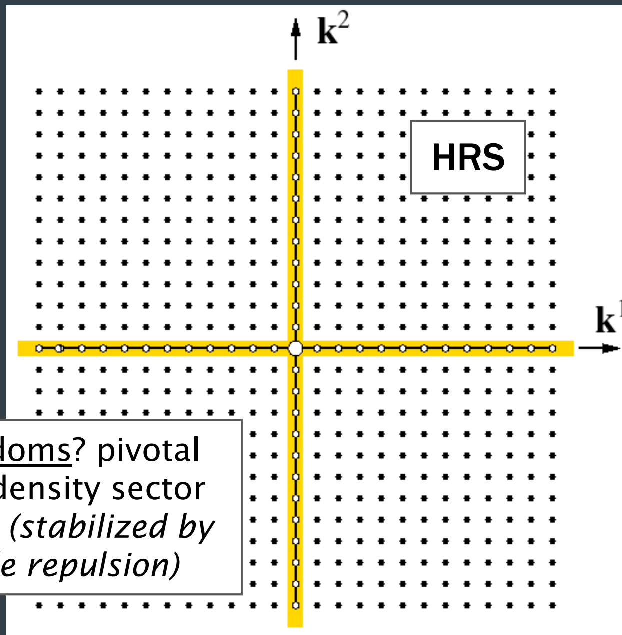
MINIMAL FIELD THEORY FOR VULCANIZED MEDIA

$$\mathcal{F} \sim \int d\widehat{x} \left\{ -\frac{\tau}{2} \Omega(\widehat{x})^2 + \frac{\xi_0^2}{2} |\widehat{\nabla}\Omega(\widehat{x})|^2 - \frac{g}{3!} \Omega(\widehat{x})^3 \right\}$$

- hats? $\widehat{x} \equiv (\mathbf{x}^0, \mathbf{x}^1, \ldots, \mathbf{x}^n)$
- excess constraint density τ
- polymer size ξ_0
- nonlinear coupling g

- derivation? microscopics or symmetries & length-scales

- looks simple, but…



<u>critical freedoms</u>? pivotal removal of density sector fluctuations *(stabilized by inter-particle repulsion)*

- Where does this minimal field theory come from?

ORIGINS: WHAT IS VULCANIZATION?

Hayward 1838, Goodyear 1839

Micro implications: length- & time-scales? symmetry?

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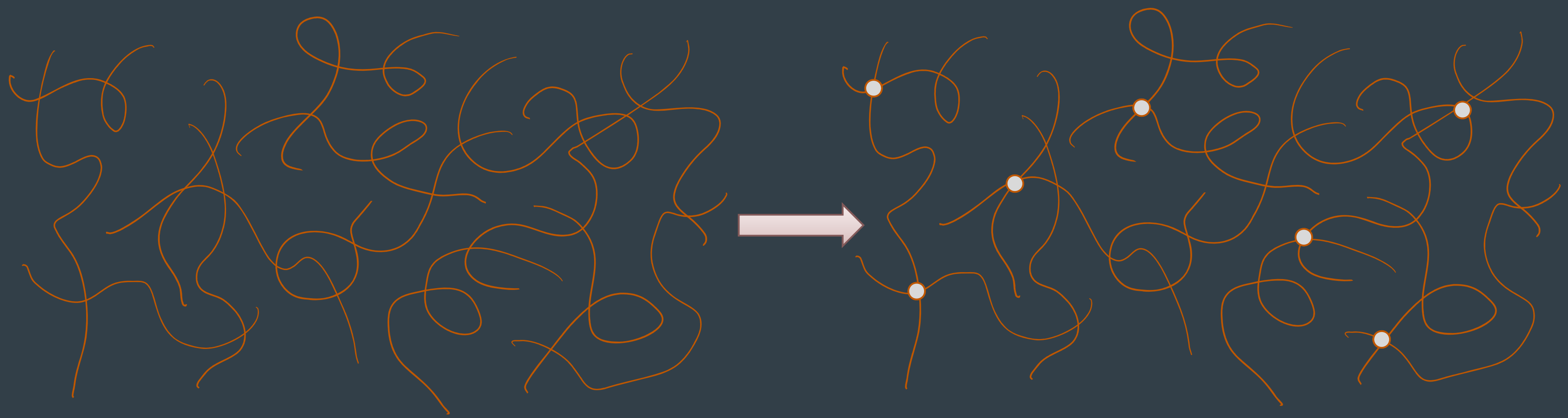
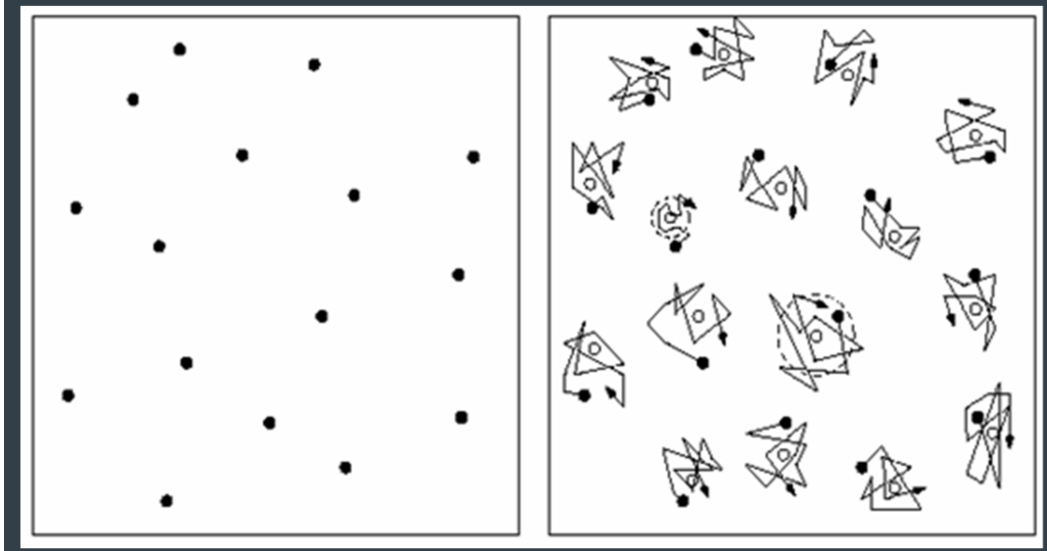
ORIGINS: WHAT ARE VULCANIZED MEDIA?

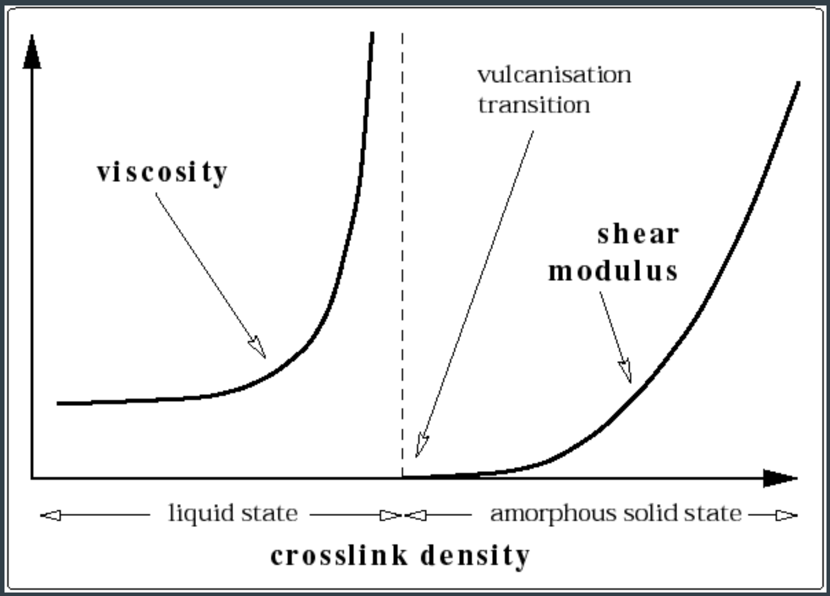
WHAT STARTS HERE CHANGES THE WORLD



NATURE'S SIMPLEST AMORPHOUS SOLIDS

ORIGINS: WHAT TRANSITION IS INVOLVED





- Micro & macro implications
- Attendant concepts: percolation, localization, cts SSB, rigidity

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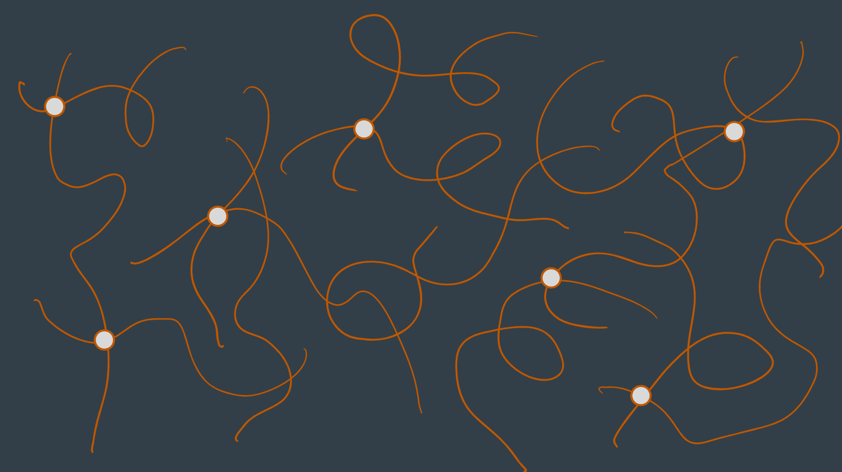
ORIGINS: ORDER PARAMETER & ITS MEANING

WHAT STARTS HERE CHANGES THE WORLD

Why are there 1+n replicas in $\mathcal{Q}(\underline{r}^0, \underline{r}^1, \ldots, \underline{r}^n)$?



Constraint distribution

(Deam & Edwards 1975)

$$\sim \frac{\mu^M}{M!}\, Z(M \text{ constraints})$$

[μ controls xl # statistics]

ORIGINS: ORDER PARAMETER & ITS MEANING

Why are they arguments in $\Omega(\underline{r}^0, \underline{r}^1, \dots, \underline{r}^n)$?

partition function

- random constraints (location & number)

$$Z \sim \int_{\text{Config}} e^{-H} \prod \delta(\text{constraints})$$

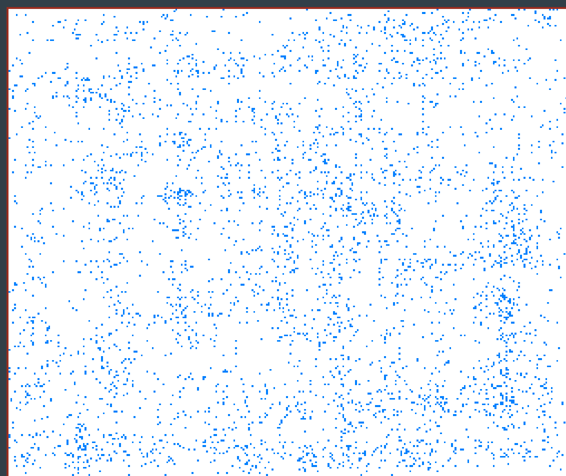
- Replicating

$$Z^n \sim \int_{\text{Config \#1}} \cdots \int_{\cdots \#n} e^{-\sum_1^n H^\alpha} \prod_1^n \delta(\text{constraints}^\alpha)$$

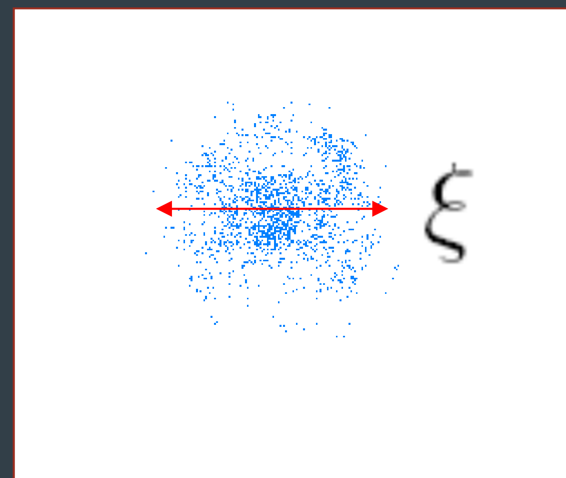
- What information does this order parameter encode?

ORIGINS: ORDER PARAMETER & ITS MEANING

One particle…



$$\langle \exp i\mathbf{k}\cdot\mathbf{R}\rangle = 0$$



$$\langle \exp i\mathbf{k}\cdot\mathbf{R}\rangle \approx \exp i\mathbf{k}\cdot\langle\mathbf{R}\rangle \exp(-k^2\xi^2/2)$$

- random mean position
- random localization length

ORIGINS: ORDER PARAMETER & ITS MEANING

Many particles…

OP meaning:

$$\Omega\left(\mathbf{k}^0, \mathbf{k}^1, \ldots, \mathbf{k}^n\right) = \left[\frac{1}{N}\sum_{j=1}^{N}\langle \mathrm{e}^{i\mathbf{k}^0\cdot\mathbf{R}_j}\rangle\langle \mathrm{e}^{i\mathbf{k}^1\cdot\mathbf{R}_j}\rangle \cdots \langle \mathrm{e}^{i\mathbf{k}^n\cdot\mathbf{R}_j}\rangle\right]$$

OP value:

$$q\,\delta_{\sum_{\alpha=0}^{g} k^\alpha,\mathbf{0}} \int \mathrm{d}\xi\; p(\xi)\exp\left(-\frac{\xi^2}{2}\sum_{\alpha=0}^{g}|k^\alpha|^2\right)$$

localized fraction q

localization length distribution p

$$p(\xi) \equiv \left[(qN)^{-1}\sum_{j\text{ loc.}}\delta^{(1)}(\xi-\xi_j)\right]$$

- Implications – conventional analysis of an unconventional field theory

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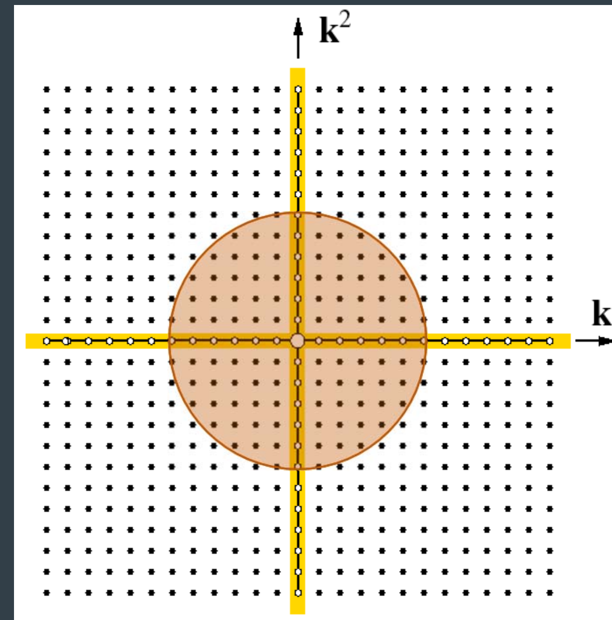
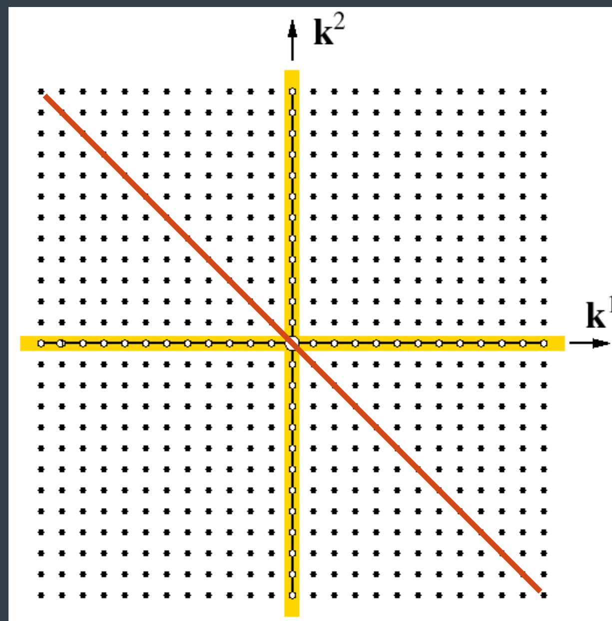
IMPLICATIONS: CLASSICAL TRANSITION

WHAT STARTS HERE CHANGES THE WORLD

$$\mathcal{F} \sim \int d\widehat{x} \left\{ -\frac{\mathcal{T}}{2}\Omega(\widehat{x})^2 + \frac{\xi_0^2}{2}|\widehat{\nabla}\Omega(\widehat{x})|^2 - \frac{g}{3!}\Omega(\widehat{x})^3 \right\}$$



- unstable modes at +ve $\mathcal{T}$



- condensation at +ve $\mathcal{T}$

- hats $\widehat{x} \equiv (\mathrm{x}^0, \mathrm{x}^1, \ldots, \mathrm{x}^n)$
- excess constraint density $\mathcal{T}$
- polymer size ξ_0
- nonlinear coupling g

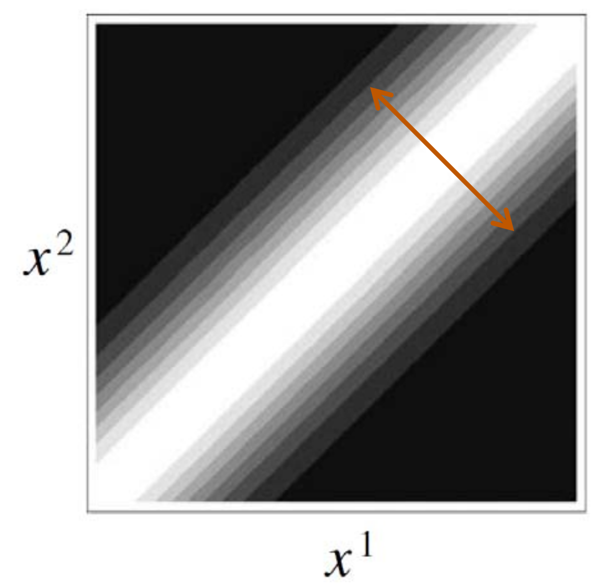
IMPLICATIONS: CLASSICAL TRANSITION

$$\mathcal{F} \sim \int d\widehat{x} \left\{ -\frac{\tau}{2}\Omega(\widehat{x})^2 + \frac{\xi_0^2}{2}|\widehat{\nabla}\Omega(\widehat{x})|^2 - \frac{g}{3!}\Omega(\widehat{x})^3 \right\}$$

- subcritical crosslink density
  - all delocalized $\boxed{\Omega(\widehat{x}) = 0}$
- supercritical…

$$\Omega(\widehat{x}) \sim q \int dz \int d\xi\, \wp(\xi)\, \mathrm{e}^{-\frac{1}{2\xi^2}\sum_{\alpha=0}^{n}|x^\alpha - z|^2}$$

  - *ripple* in replicated space (glide?)
  - 'molecular' bound state
  - random localization:
    peak gives *fraction*, shape gives *distribution*
  - so encodes statistical information about
    heterogeneity of random solid state *structure*



- hats $\widehat{x} \equiv (\mathbf{x}^0, \mathbf{x}^1, \ldots, \mathbf{x}^n)$
- excess constraint density τ
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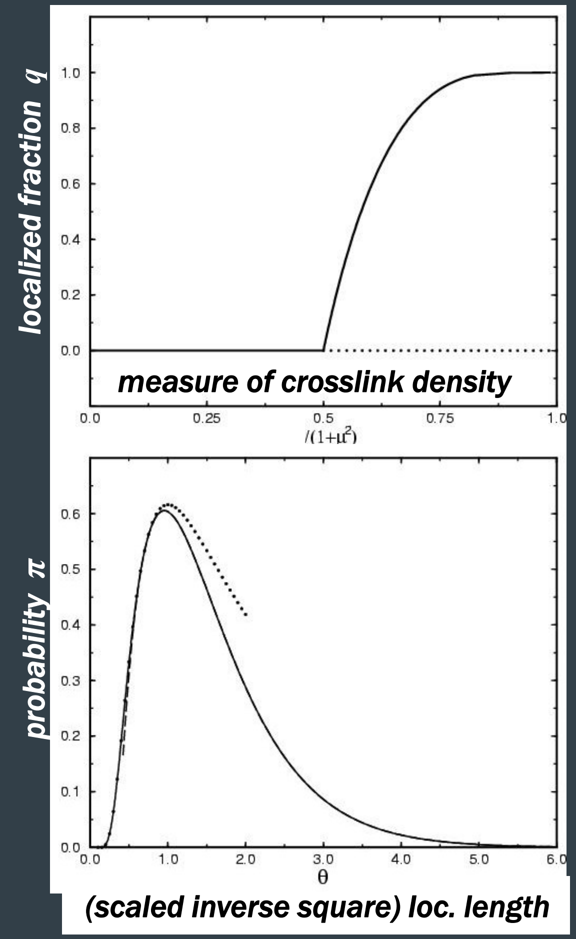
IMPLICATIONS: CLASSICAL TRANSITION

- localized fraction q
  - recovers Erdős-Rényi
  - classical percolation exponent

- distribution of localization lengths
  - predicts data collapse
  - universal scaling form
  - elementary derivation?

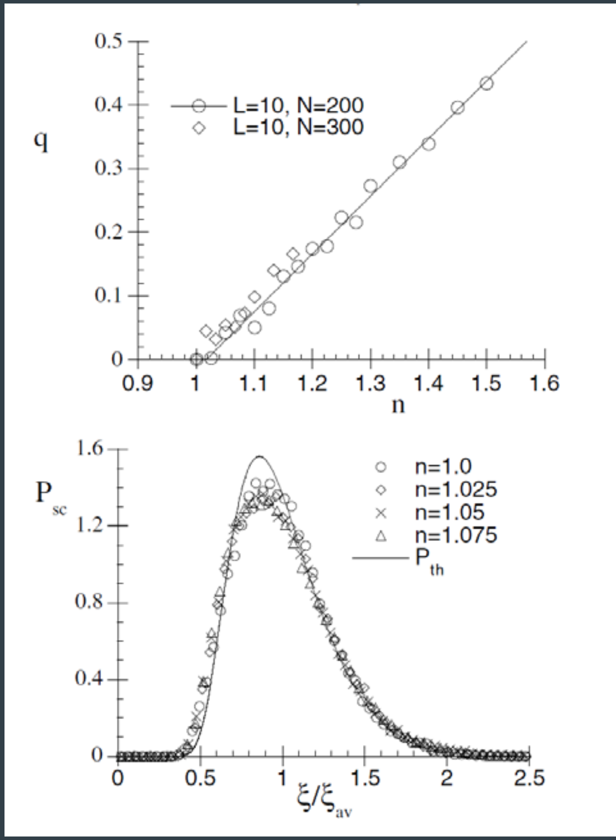
$$q = 2\epsilon/3$$
$$p(\xi) = (4/\epsilon\xi^3)\pi(2/\epsilon\xi^2)$$

$$\frac{\theta^2}{2}\frac{\mathrm{d}\pi}{\mathrm{d}\theta} = (1-\theta)\pi - \int_0^{\theta} \mathrm{d}\theta'\ \pi(\theta')\pi(\theta-\theta')$$

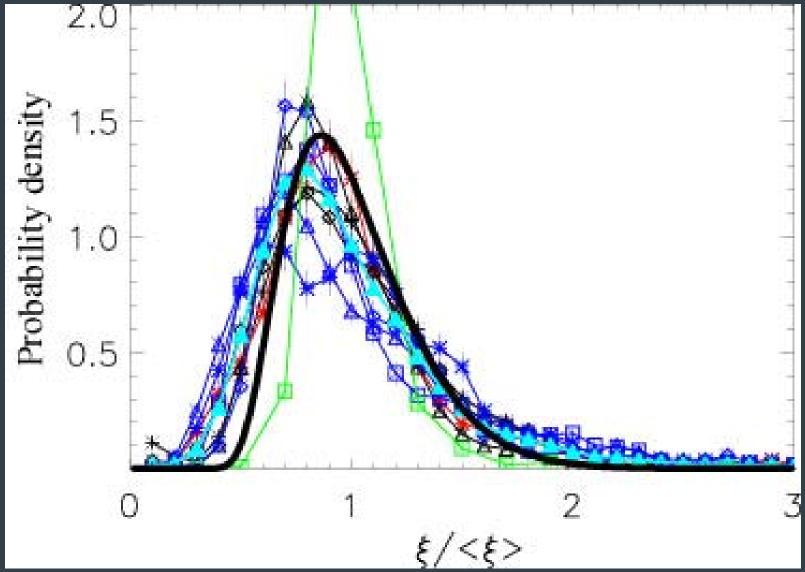


SIMULATIONS & EXPERIMENTS

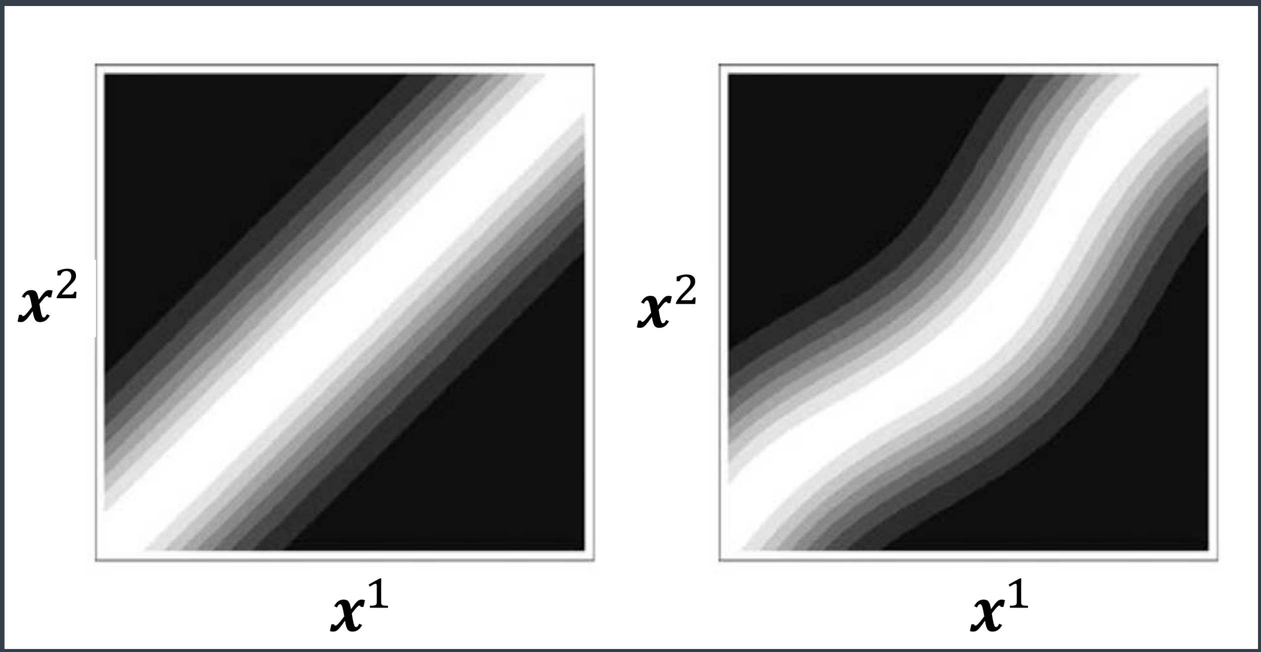
Barsky & Plischke simulations



Dinsmore group experiments



GOLDSTONE EXCITATIONS & ELASTICITY



D-dimensional sheet in $(1+n)D$ dimensions

- their energetics describes elasticity
  - average stiffness; its power-law criticality
  - scale-dep elasticity probes DLL
  - spatial heterogeneity
  - LCD = 2
  - NLSM in 2+ε ? D-brane-like; role of incompress.
  - cf liquid-gas interface, vortices,…

IMPLICATIONS: BEYOND CLASSICAL

$$\delta\tau \sim (L/\ell)^{-(d-2)/(6-d)}\varphi^{-2/(6-d)}$$

- **Ginzburg – de Gennes (1977) criterion**
  - lcd 2, ucd 6
  - narrow for long, dense chains
- **critical state** [see also Stenull-Janssen, 2001]
  - percolation questions get exactly percolation answers (flow eqs, exponents)
  - vulcanization field theory <u>contains</u> percolation +…
  - hints of SUSY cancelations
- **field correlations**
  - liquid: mutual localization (aka clusters)
  - solid: not easy – correlations in structural heterogeneity

WHY NATURE'S SIMPLEST AMORPHOUS SOLIDS

- **glass vs vulcanized matter**
  - both settings – dynamics constrained
- **glass constraints**
  - feedback-generated: structure → dynamics → structure →…
  - no clear time-scale separation, essentially dynamical
- **vulcanized matter constraints**
  - extrinsic, time-scale separation admits eqm stat mech
  - liquid state <u>truly</u> frozen in
- **symmetry & length-scale arguments supply LW effective hamiltonian**
  - unconventional theory, conventional analysis
  - embraces distinct concepts: localization, percolation & rigidity
  - replicas seens as a smart encoding of heterogeneity

OPEN CHALLENGES

- **additional freedoms – some work done**
  - liquid crystal elastomers, blends
- **fluctuations in the rigid state**
  - near-critical DLL, LL correlations
- **VM near 2D**
- **SUSY connections**
  - body & soul
- **experiments**
  - criticality, heterogeneity, QENS, micro-rheology
- **dynamics**
  - minimal model?

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