# Universality and Renormalization Group for Weyl semimetals

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- It is due to Nielsen and Ninomiya (1983) the proposal that the Adler-Bell-Jackiw anomaly has a counterpart in the transport properties of such materials.
- Observation of the analogue of the chiral anomaly in magneto-conductance experiments have been reported (see e.g. Ong et al Science (2015))

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- The applicability to Weyl semimetals is unclear: the corrections due to the lattice, even if irrelelvant, produce finite corrections.
- It is indeed well known that irrelevant terms play a crucial role in transport.

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- It is not restrictive to consider a simple model, like the one with hopping Hamiltonian (Delplace Carperntier EPL 2010), $k_\pm = (k_1 \pm k_2)/2$

$$h_0(\vec{k}) = \begin{pmatrix} t_\perp \cos k_3 - \mu + t' + \alpha(\vec{k}) & t(\sin k_+ + i \sin k_-) \\ -t(\sin k_+ - i \sin k_-) & t_\perp \cos k_3 - \mu + t' + \alpha(\vec{k}) \end{pmatrix}$$

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- It describes fermions on a cubic lattice with suitable magnetic flux (zero magnetic flux in each cell and inv symmetry) : $t'$ nearest planar, $t$ next to nearest planar hopping with suitable phases, $t_3$ between planes and $\mu$ describes the difference of densities. If $|\mu - t'|/t_\perp$ there are 2 Fermi points $(0, 0, \pm p_F)$ with $t_\perp \cos p_F = \mu - t$.

# Interacting lattice Weyl semimetal

- We add an interaction so that the interating model is

$$H = \int d\vec{k}\psi^+_{\vec{k}} h_0(\vec{k})\psi^-_{\vec{k}} + U \int d\vec{p}\hat{v}(\vec{p})\rho_{\vec{p}}\rho_{-\vec{p}}$$

where $\rho_{\vec{p}} = \int \frac{d\vec{k}}{(2\pi)^3}\hat{\psi}^+_{\vec{k}+\vec{p}}\hat{\psi}^-_{\vec{k}}$ is the local density, $\hat{v}(\vec{p})$ is a short range potential. and $\psi^\pm = (a^\pm, b^\pm)$.

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- In absence of interaction the 2-point function $g(\mathbf{x})$ close to the Weyl point has the form, if $\mathbf{k} = \mathbf{k}' + \mathbf{p}^{\omega}_{F}$, $\omega = \pm$,

$$\frac{\chi(\mathbf{k}')}{Z} \begin{pmatrix} -ik_0 + v\omega v^0_3 k'_3 & v^0_+(k_+ - ik_+) \\ v^0_+(k_+ + ik_-) & -ik_0 - v^0_3 \omega k'_3 \end{pmatrix}^{-1} (1 + R(\mathbf{k}))$$

with $Z = 1$, $v^0_3 = t_\perp \sin p_F$, $v^0_\pm = t$.

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- The model admits an effective description in term of massless Dirac particles with a loca interaction. This QFT model is invariant under a global $\psi^\pm \to e^{\pm i\alpha}\psi^\pm$ and chiral $\psi^\pm \to e^{\pm i\gamma_5\alpha}\psi^\pm$; the second symmetry is not true in the lattice model.

# Axial density

- We introduce the following operator

$$\hat{\rho}^5_{\vec{p}} = \int \frac{d\vec{k}}{(2\pi)^3} \frac{\sin k_3}{\bar{Z}} \hat{\psi}^+_{\vec{k}+\vec{p}} \hat{\psi}^-_{\vec{k}}$$

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- We can define in a similar way the analogue of the axial current

# Background e.m. field

- Response of the system to background e.m. field; in particular $A_2 = Bx_1$ and $\partial_0 A_3 = E$ and $A_0 = A_1 = 0$.

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- Grassmann integral representation; the e.m. field is introduced via the Peierls substitution, by modifying the hopping parameter along the bond $(\vec{x}, \vec{x} + \vec{\delta})$ as $U_{\vec{x}, \vec{x}+\vec{\delta}}(\vec{A}) = e^{ie \int_0^1 \vec{\delta} \cdot \vec{A}(\vec{x} + s\vec{\delta}) ds}$.

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- Generating functional

$$e^{W(A_\mu, A^5_\mu, \phi)} = \int P(d\psi) e^{V + \int A^5_\mu j^5_\mu(A) + B(A) + (\psi, \phi)}$$

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- $B(A)$ is the interaction with the background e.m. field, $j^5_\mu(A)$ is the chiral current (depending in $A$ via the Peierls) and $\nu$ is a counterterm to fix the the Fermi momentum.

# Ward Identities

- $<>_A = \frac{\int P(d\psi) e^{V+B(A)} O}{\int P(d\psi) e^{V+B(A)}}$ and by $\Gamma_{\mu,\mu_1,..,\mu_n}$ the derivative of $W$ with respect to $A_\mu, A_{\mu_1}, ...A_{\mu_n}$

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$$ip_0 < \rho^5_{\bar{p}} >_A = ip_0 \Gamma^5_{0,\nu}(\bar{\mathbf{p}}) A_{\nu,\bar{\mathbf{p}}} +$$

$$ip_0 \int d\mathbf{p}_1 d\mathbf{p}_2 \delta(\mathbf{p}_1 + \mathbf{p}_2 = \bar{\mathbf{p}}) \Gamma^5_{0,\nu,\sigma}(\mathbf{p}_1, \mathbf{p}_2) A_{\nu,\mathbf{p}_1} A_{\sigma,\mathbf{p}_2} + R$$

where $R$ contains higher order terms. A similar expression holds for the current, with $\Gamma$ replacing $\Gamma^5$; in that case the r.h.s. would be vanishing.

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- What happens with a many body interaction?

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- It implies $\mathbf{p}_\mu \Gamma_{\mu,\mu_1,..\mu_n} = 0$ implying the conservation of the current $< \mathbf{p}_\mu j_\mu >_A = 0$ in presence of an e.m. field. Ward identies provide informations also on $\Gamma^5$; in particular
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- An important role will be played also by the vertex WI

$$p_\mu \langle \hat{j}_{\mu,\mathbf{p}} \hat{\psi}^+_{\mathbf{k}+\mathbf{p}} \hat{\psi}^-_{\mathbf{k}} \rangle = (\langle \psi^+_{\mathbf{k}+\mathbf{p}} \psi^-_{\mathbf{k}+\mathbf{p}} \rangle - \langle \psi^+_{\mathbf{k}} \psi^-_{\mathbf{k}} \rangle)$$

# Renormalization Group analysis

- We can write $\Gamma^5_{0,\nu,\sigma} = < \rho^5; j_\nu; j_\sigma > +$

$$< \partial_{A_\nu} \rho^5; j_\sigma > + < \partial_{A_\sigma} \rho^5; j_\nu > + < \partial_{A_\sigma} \partial_{A_\nu} \rho^5 >$$

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- Writing $\psi = \sum_{\omega=\pm} \sum_{h=-\infty}^{0} \psi^h_\omega$, where $\psi^h_\omega$ are living in a shell $O(2^h)$ around the Weyl points $\mathbf{p}^\omega_F$, $\omega = \pm$.

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- Integrating out the fields $\psi^0, \psi^{-1}_\pm, .., \psi^h_\pm$ one get that $e^W = \int P(d\psi^{(\leq h)}) e^{V^h}$, where $g^{\leq h}(\mathbf{k})$ is similar to $g(\mathbf{k})$ with $Z_h, v_{+,h}, v_{3,h}$ anf $V^h = \tilde{V}^h + 2^h \nu_h \sum_\omega \int d\mathbf{x} \psi^+_{\mathbf{x},\omega} \omega \psi^-_{\mathbf{x},\omega} +$

$$Z_{\mu,h} \sum_\omega \int d\mathbf{x} A_\mu j_{\mu,\omega,h} + Z^5_h \sum_\omega \int d\mathbf{x} A^5_\mu \omega j^h_{0,\omega,h} + \mathcal{R} V^h$$

  with $\mathcal{R} V^h$ contain irrelevant terms (non local or of order $\geq 4$). All the effective parameters and generically physical observables are interaction dependent (Mastropietro JPA, JSP 2013).

# Renormalization Group analysis

- The main outcome of the RG analysis is that

$$\Gamma^5_{\mu,\nu,\sigma} = \Gamma^{5,R}_{\mu,\nu,\sigma} + H_{\mu,\nu,\sigma}$$

where $\Gamma^{5,R}_{0,\nu,\sigma}$ is the chiral anomaly (triangle graph) of a relativistic theory with Dirac fermions with momentum cut-off, wave function renormalization $Z_{-\infty} = 1 + O(U^2)$, velocities $v_{i,-\infty} = v_i + O(U)$ and vertex renormalizations $Z_{\mu,-\infty} = 1 + O(U)$.

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- By an explicit computation
$(p_{1,\mu} + p_{2,\mu})\Gamma^{R,5}_{\mu,\nu,\sigma} = \frac{Z_{\nu,-\infty} Z_{\sigma,-\infty} v_{\alpha,-\infty} v_{\beta,-\infty}}{6\pi^2 v_+^2 v_3 Z^2_{-\infty}} p_{1,\alpha} p_{2,\beta} \varepsilon_{\alpha,\beta,\nu,\sigma}$.

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- MAIN POINT: $H_{0,\nu,\sigma}$ is differentiable while $\Gamma^{5,R}_{0,\nu,\sigma}$ is only continuous.
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$(p_{1,\mu} + p_{2,\mu})\Gamma^{R,5}_{\mu,\nu,\sigma} = \frac{Z_{\nu,-\infty} Z_{\sigma,-\infty} v_{\alpha,-\infty} v_{\beta,-\infty}}{6\pi^2 v_+^2 v_3 Z^2_{-\infty}} p_{1,\alpha} p_{2,\beta} \varepsilon_{\alpha,\beta,\nu,\sigma}$.
- Note that $p_{1,\nu}\Gamma^{R,5}_{\mu,\nu,\sigma}$ is also non vanishing.

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# Universality

**Theorem**(Giuliani Mastropietro Porta)*For* $U$ *small enough and chosen* $\nu = O(\lambda), \tilde{Z} = 1 + O(\lambda)$ *then the 2-point function has renormalized parameters*

$$v_3 = v_{3,0} + a_3 U + O(U^2)) \quad v_\pm = v_{\pm 0} + a_\pm U + O(U^2)$$

*and* $Z = 1 + O(U^2)$ *with* $a_3, a_\pm$ *non vanishing coefficients. Moreover* $\Gamma^5_{\mu,\nu} = O(p^2)$ *and*

$$(p_1 + p_2)_\mu \Gamma^5_{\mu,\nu,\sigma} = \frac{1}{2\pi^2} p_{1,\alpha} p_{2,\beta} \varepsilon_{\alpha,\beta,\nu,\sigma} + O(p^2)$$

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All the physical parameters are renormalized by the interaction and depend on the all the microscopic details; however the the variation of the difference of density between the Fermi points is $\frac{1}{2\pi^2}\vec{E}\vec{B}$, in presence of a magnetic and electric field.

# Conclusions

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- It would be important to extend to Coulomb interactions.