

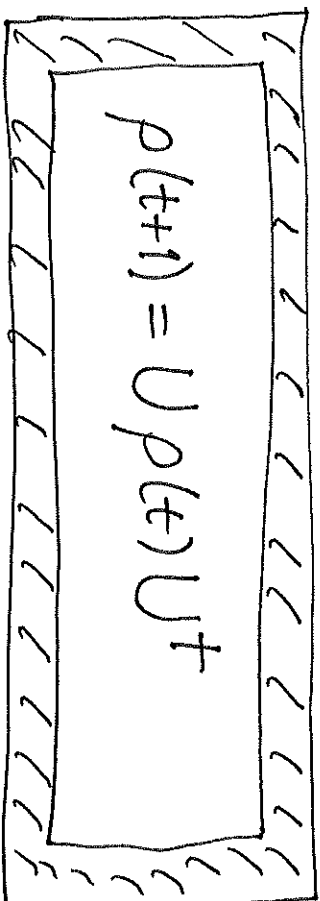
The { quantum glass transition
dynamical quantum phase transition } between the
many-body localized (MBL) phase and the thermal phase.

— with Vadim Oganesyan and many others....
(CUNY)

A review: Nandkishore and Huse, arXiv:1404.0686
Ann. Rev. Cond. Matt. Phys. 6, 15 (2015).

Consider:

A closed many-body quantum system:
(e.g. of ultracold atoms, q-bits, ...)



one case of
interest: time-
independent H :
 $U = e^{-iH}$

ρ : quantum state (density operator) on a
Hilbert space that is the product over local
Hilbert spaces.

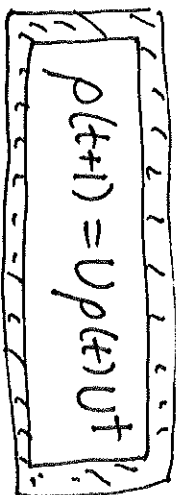
Example: L two-state system (spins-1/2, q-bits):
Hilbert space dimension $= 2^L$

$\rightarrow$ Short-range interactions: U is a local unitary

- Locality and
- Unitarity

ρ : highly-excited
highly-entangled

U : strong interactions



$$\rho(t+1) = U \rho(t) U^\dagger$$

and strongly-interacting

Long-time dynamics of such a highly-excited many-body system?

Two generic possibilities: (each stable to small local changes to U)

1) Thermalization: System is a reservoir and brings its subsystems to a state that maximizes entropy (thermodynamic equilibrium) (possibly subject to constraints on total energy...)

2) Many-body localization: System fails to be a reservoir and stays Anderson localized "near" its initial state. A "glass".

This is a "new" type of integrable system with U -specific localized conserved operators. (" ℓ -bits")

3) and a well-studied set of special cases:

"traditional" integrable systems.

(not considered again today)

An Example: Ising chain with longitudinal and transverse fields:

$$H = \sum_{\text{sites } i=1}^L \left[\sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x \right]$$

$\swarrow$ static + random $\nearrow \vec{\sigma}_i^z : \text{spins } -\frac{1}{2}$

At zero transverse field, $\Gamma=0$, this is trivially localized.

Localized conserved operators are the $\{ \sigma_i^z \}$.

They and H all mutually commute.

Turn on small Γ : system remains localized. Conserved operators need to be "dressed" but remain localized $\{ \tau_i^z \}$ (Imbrie 2014) ("2-bits")

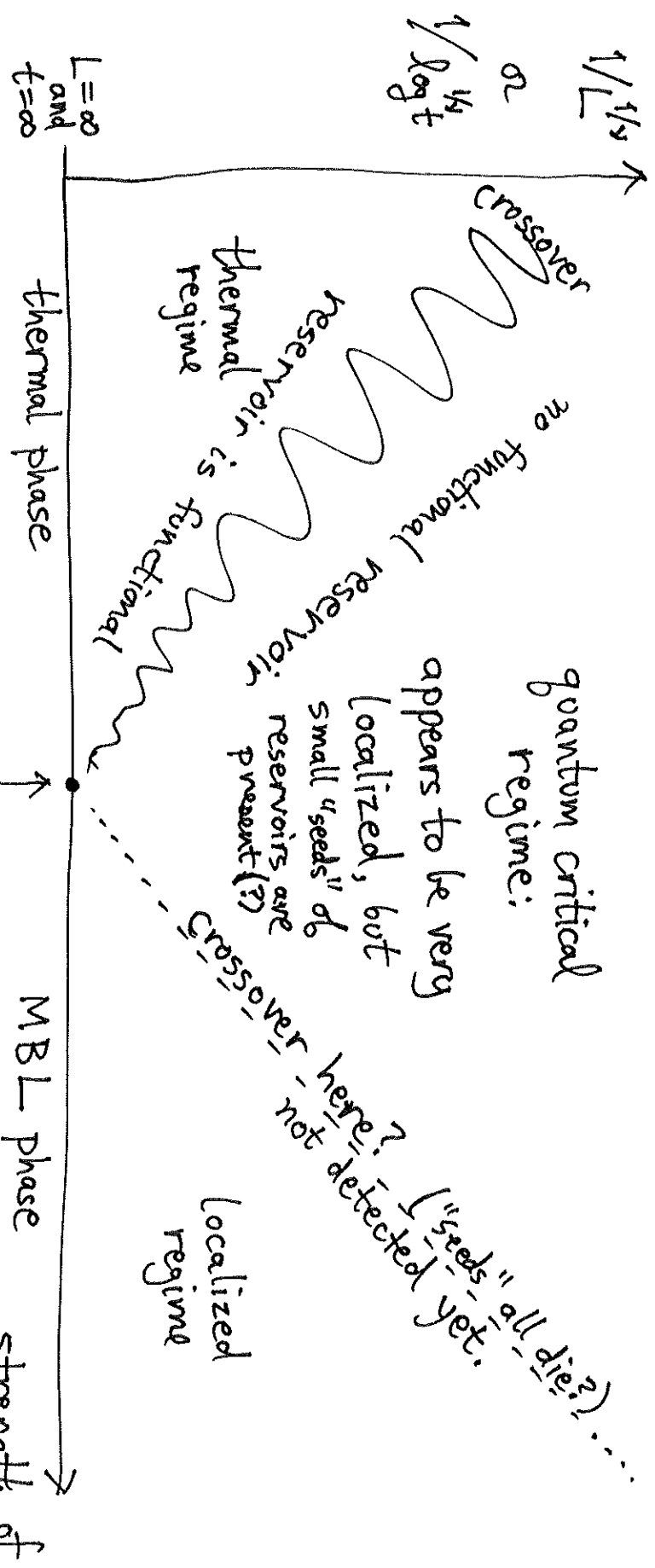
System thermalizes for larger Γ if disorder is not too strong. H., Nandkishore, Oganesyan 2014

This thermal-MBL phase diagram has been looked at

numerically for many 1D models by exact diagonalization.

1D bosons: Aleiner, Altshuler, Shlyapnikov, arXiv:0910.4534

Schematic phase diagram with finite-size or finite-time effects. 1D short range models (many works)



From studies of:

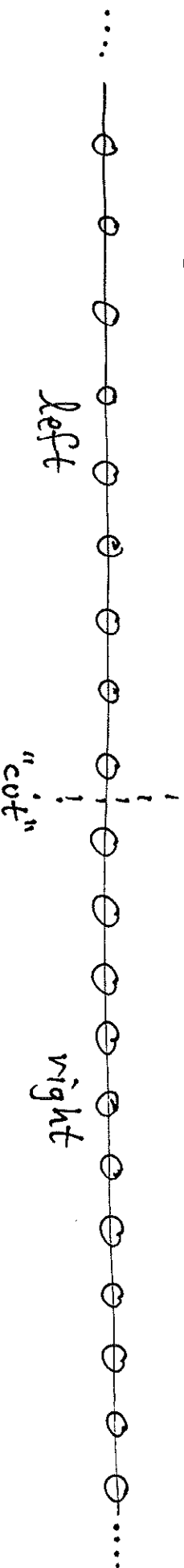
- dynamics
- level statistics
- entanglement in exact eigenstates

Mostly from chains of length 10 to 20 spins.

This picture of phase transition is particularly inspired by:

Trithap Devakul + Rajiv Singh, PRL 2015
(now Princeton) (Davis)

Infinite spin chain with static random fields: $H = \sum_i (h_i \sigma_i^z + \vec{\sigma}_i \cdot \vec{\sigma}_{i+1})$



Get exact many-spin eigenstates and calculate entanglement between left and right

Do graphical expansion for this + average over random fields and all eigenstates. ("Linked cluster expansion")

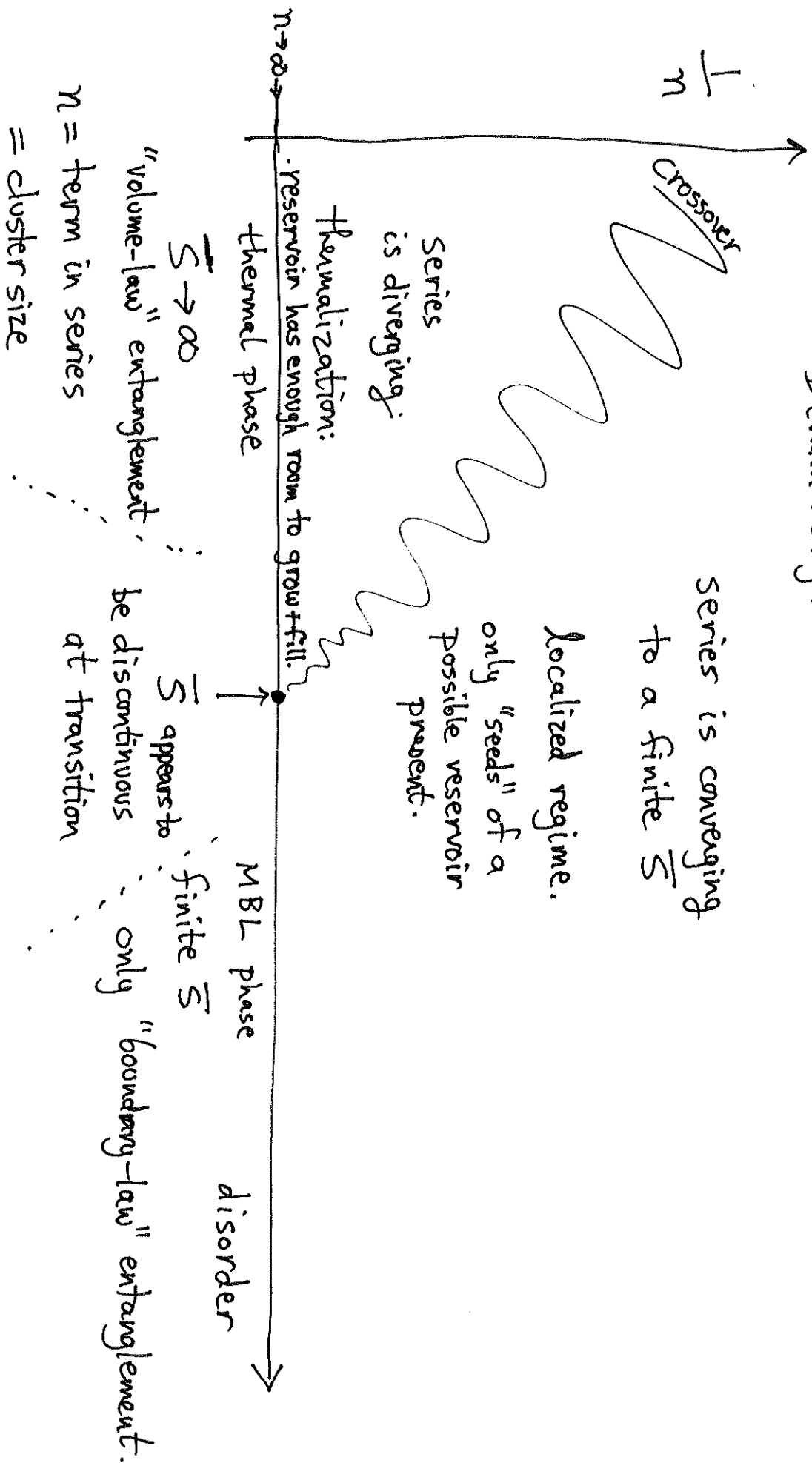
Get series for ^{average} entanglement entropy: $\overline{S} = S_0 + S_1 + S_2 + S_3 + \dots$

← first nonzero term.

MBL phase: sum converges.

Thermal phase: sum diverges.

subscript is cluster size.



Transition appears to be some "hybrid" of continuous + discontinuous.

SUMMARY

MBL-to-thermal phase transition:

The "emergence" of thermodynamic equilibrium.

A new class of phase transitions: poorly understood.

The beginnings of an understanding are emerging.