

Dynamics of Dumb and Less Dumb Foraging

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*120th Statistical Mechanics Meeting
Rutgers University, December 16, 2018*

Question: ¿ How long can a dumb (but cute) forager live?



MacArthur & Pianka (1966)

Charnov (1976)

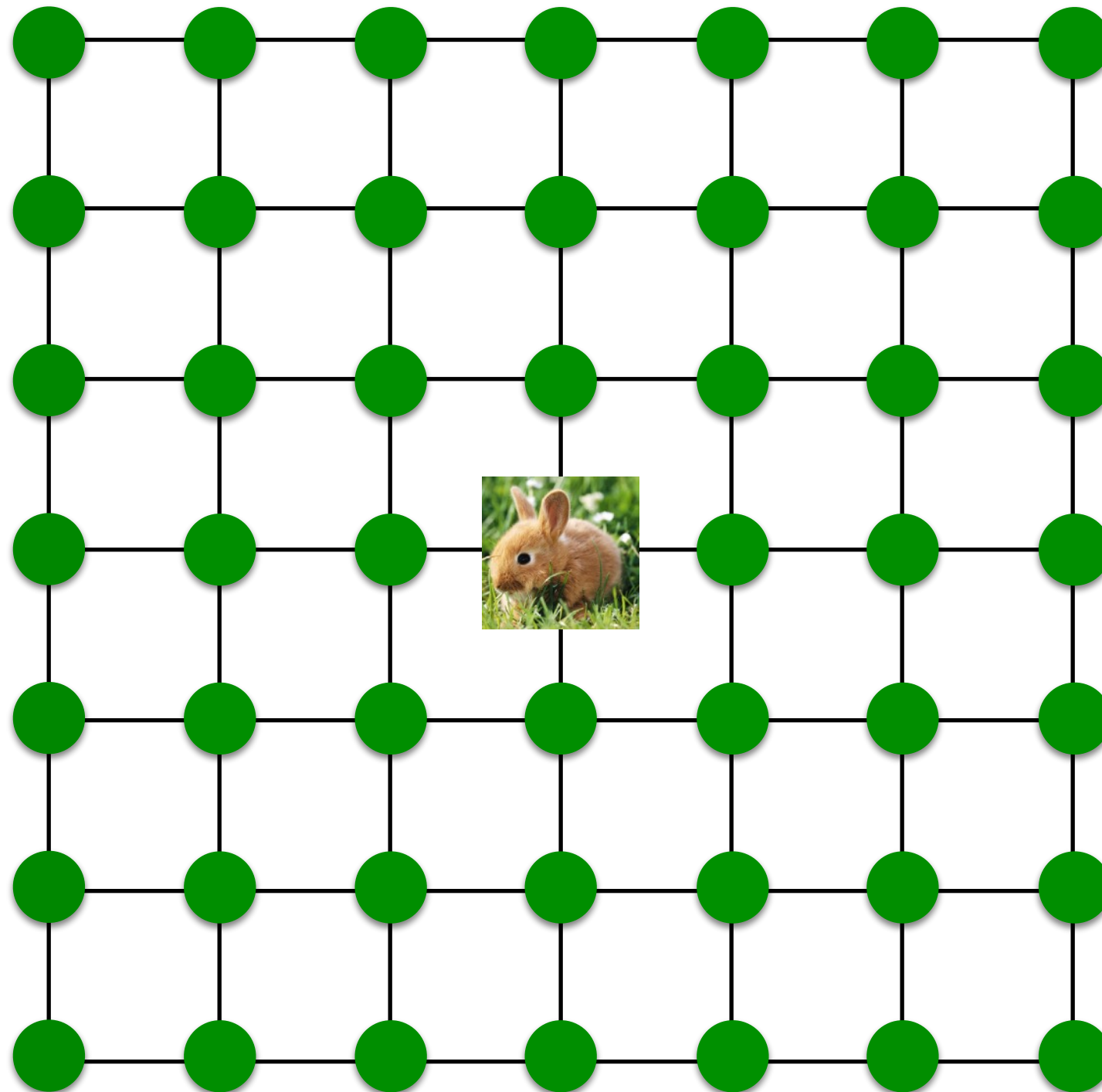
Pyle et al (1977)

Stephens & Krebs (1986)



Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



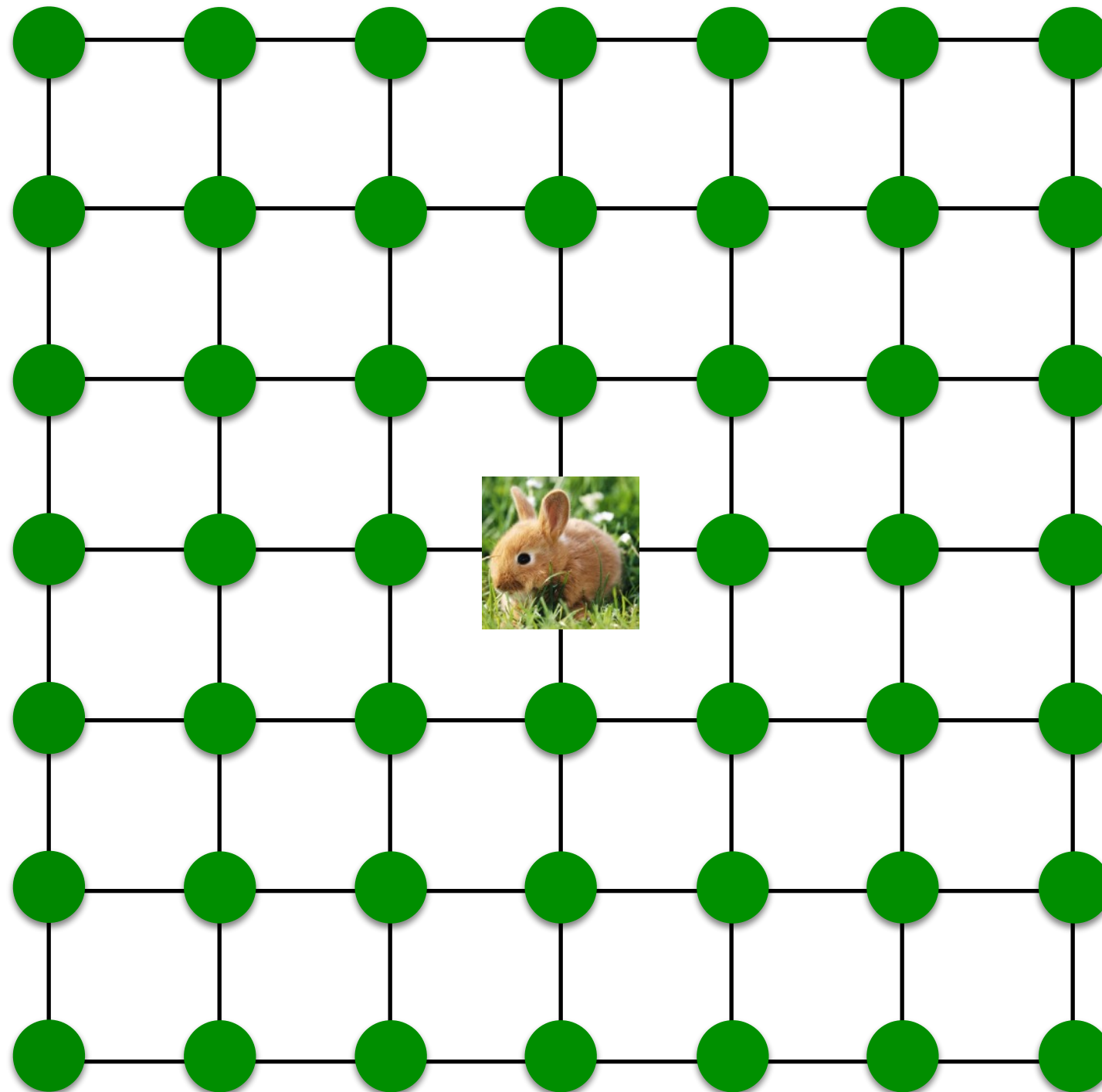
$$S = 6$$

time = 0

time left = 6

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



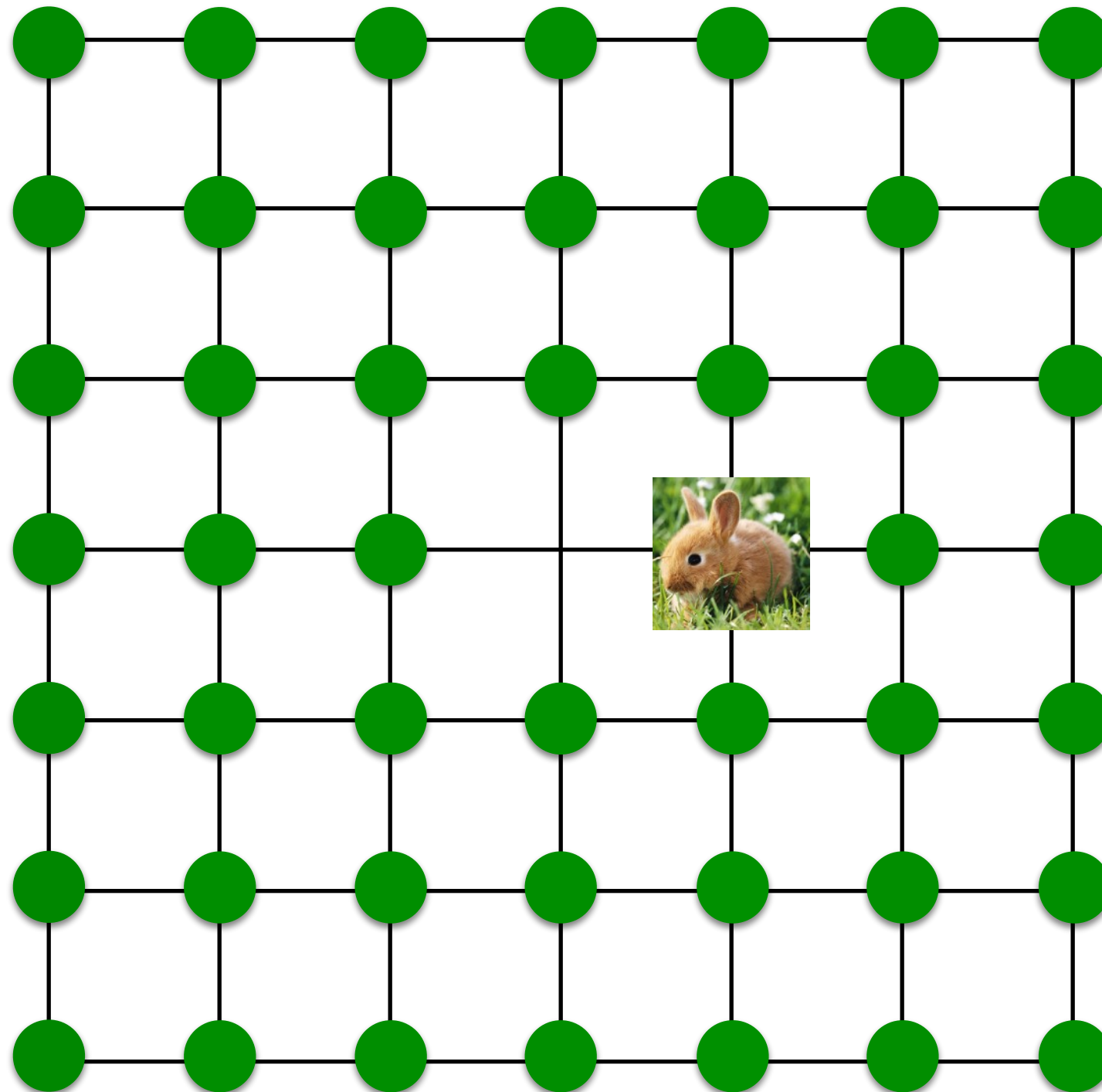
$$S = 6$$

time = 0

time left = 6

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



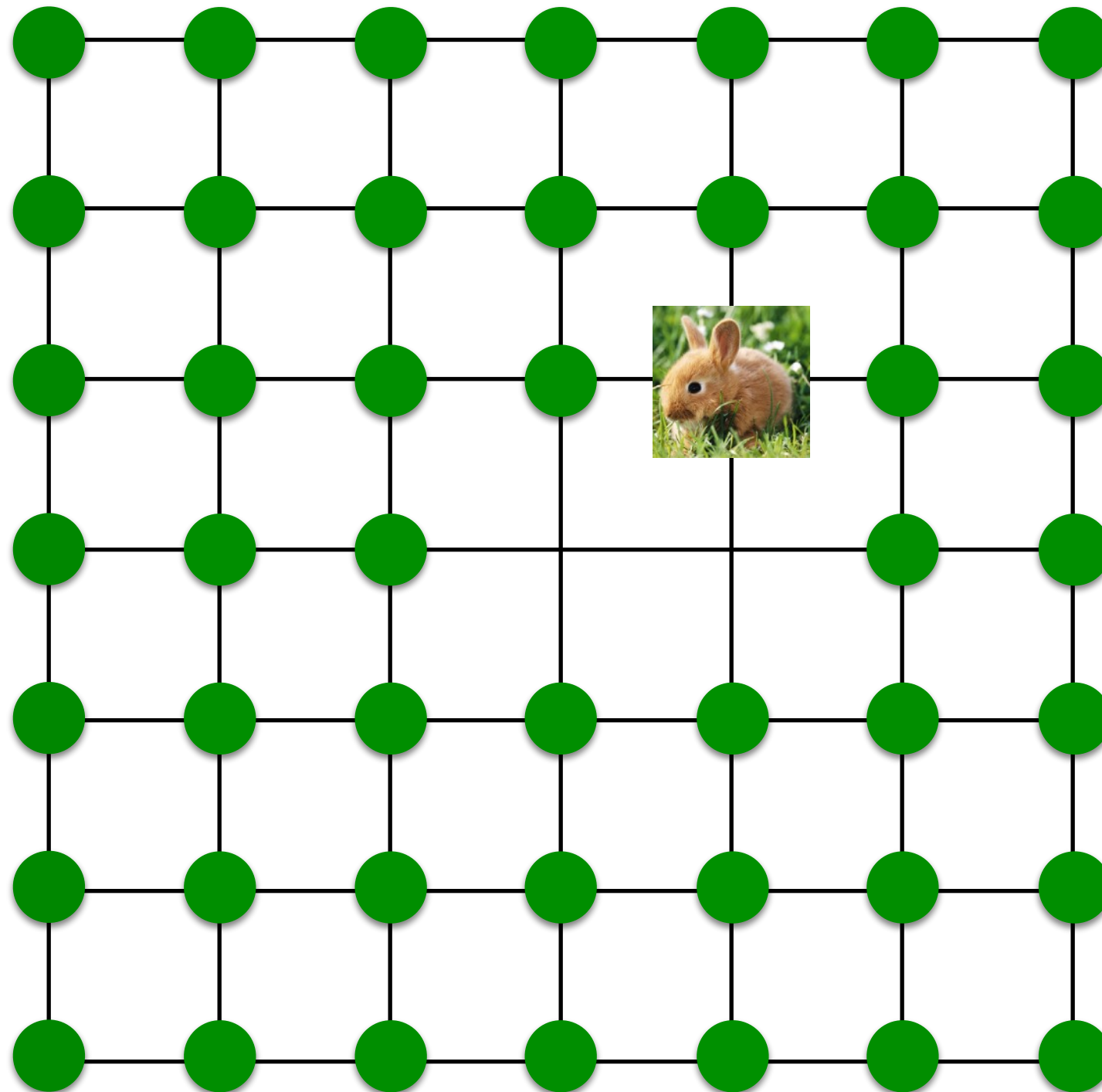
$$S = 6$$

time = 1

time left = 6

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



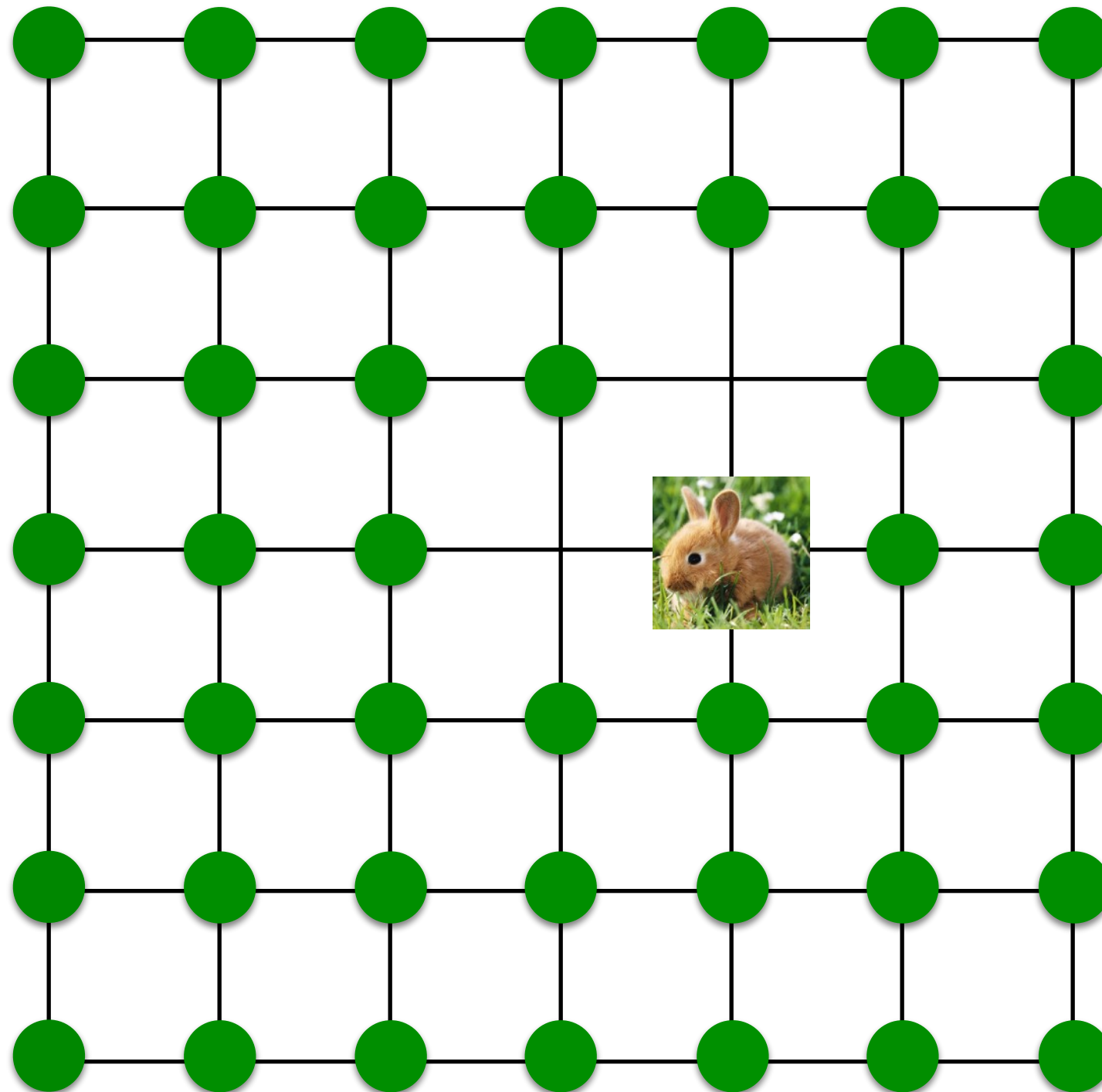
$$S = 6$$

time = 2

time left = 6

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



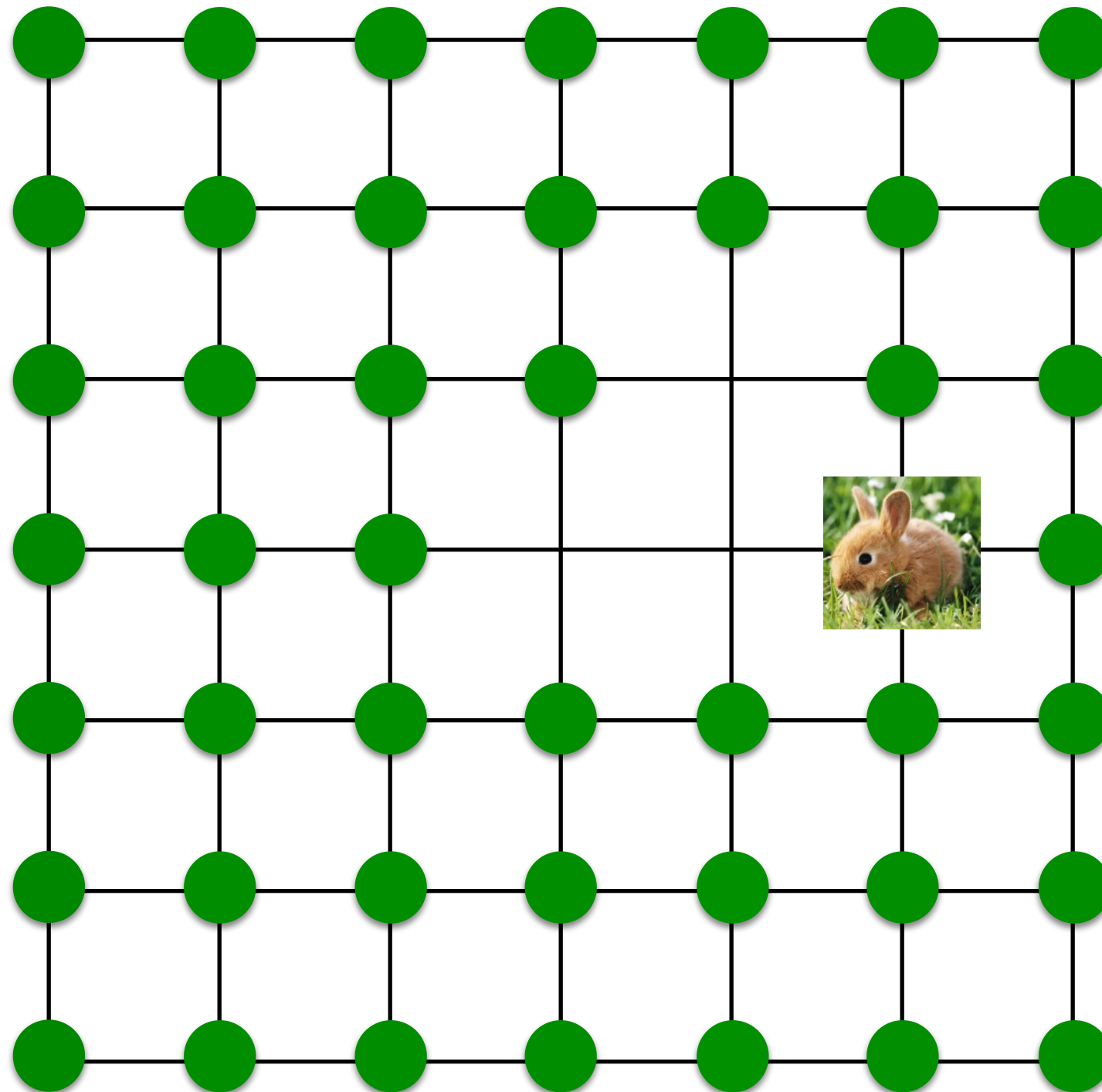
$$S = 6$$

time = 3

time left = 5

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



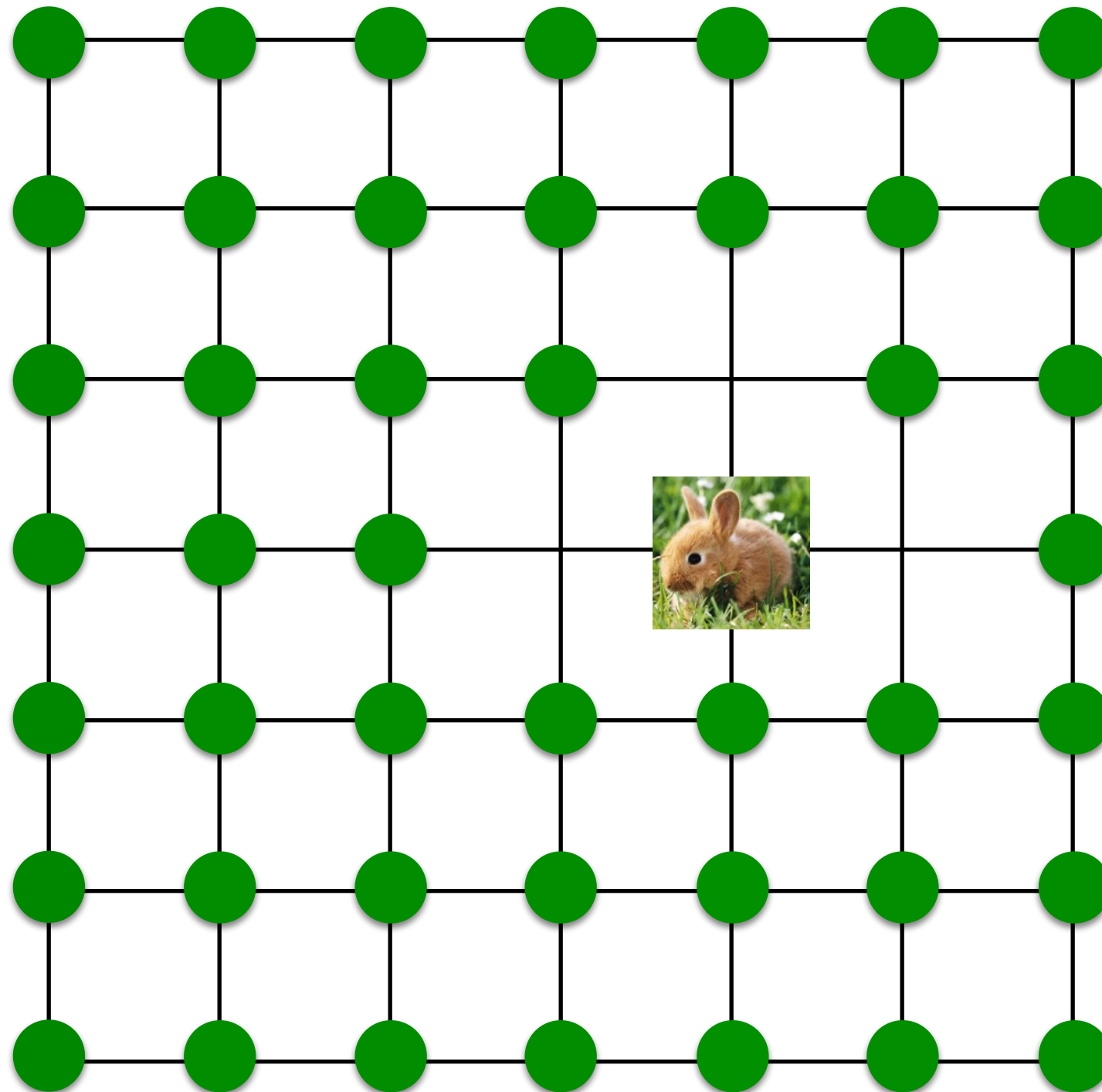
$$S = 6$$

time = 4

time left = 6

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



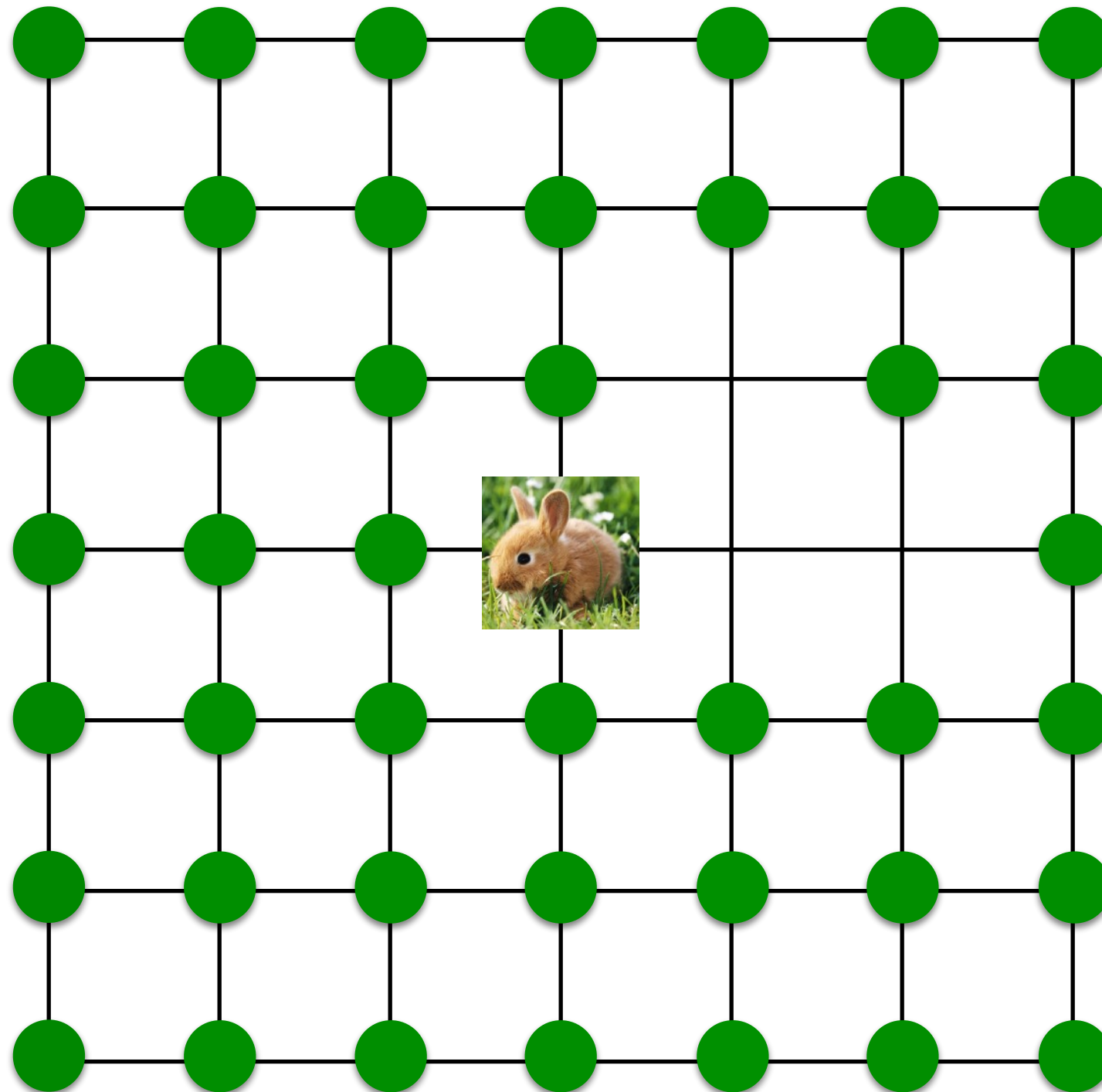
$$S = 6$$

time = 5

time left = 5

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



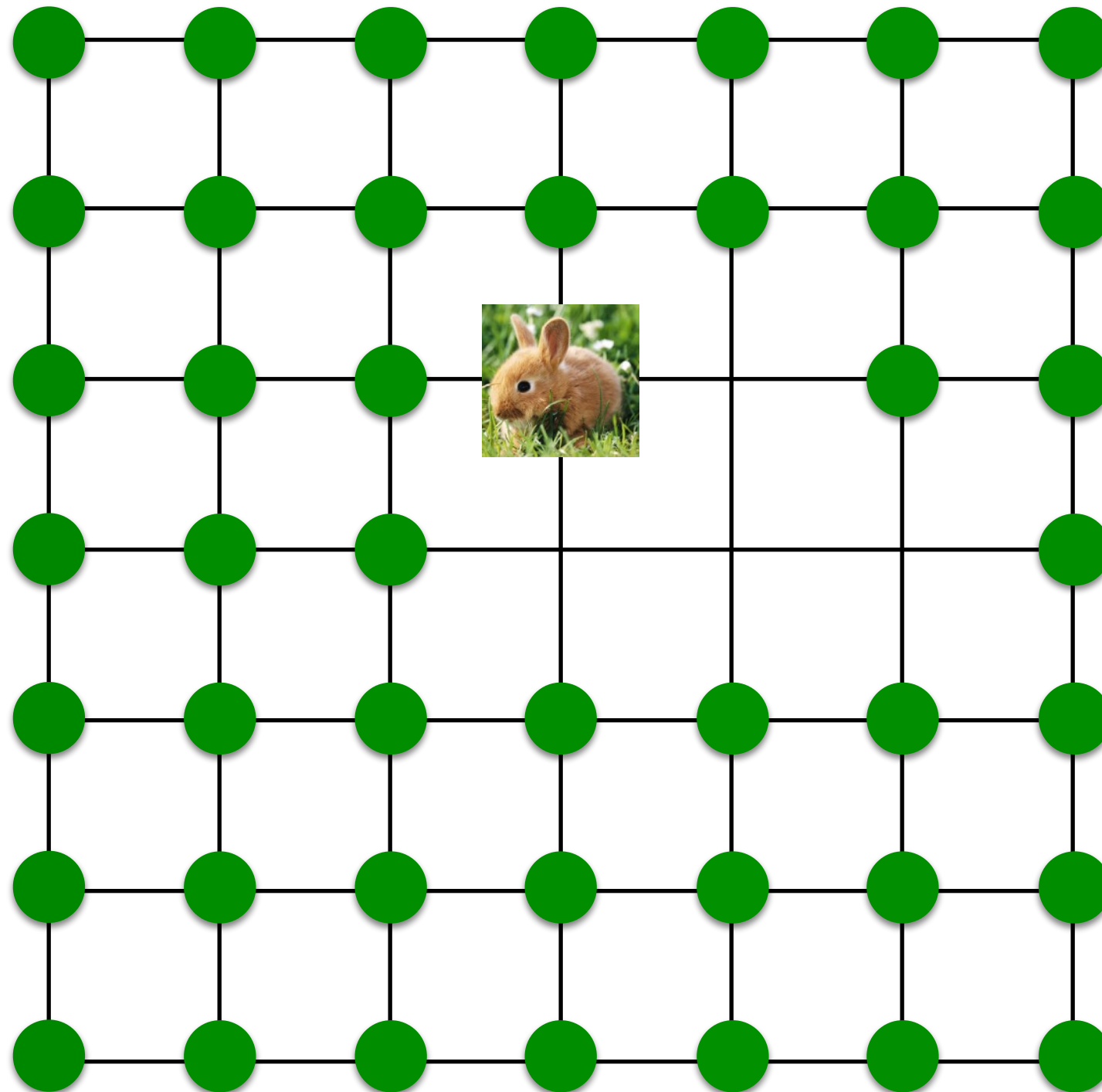
$$S = 6$$

time = 6

time left = 4

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



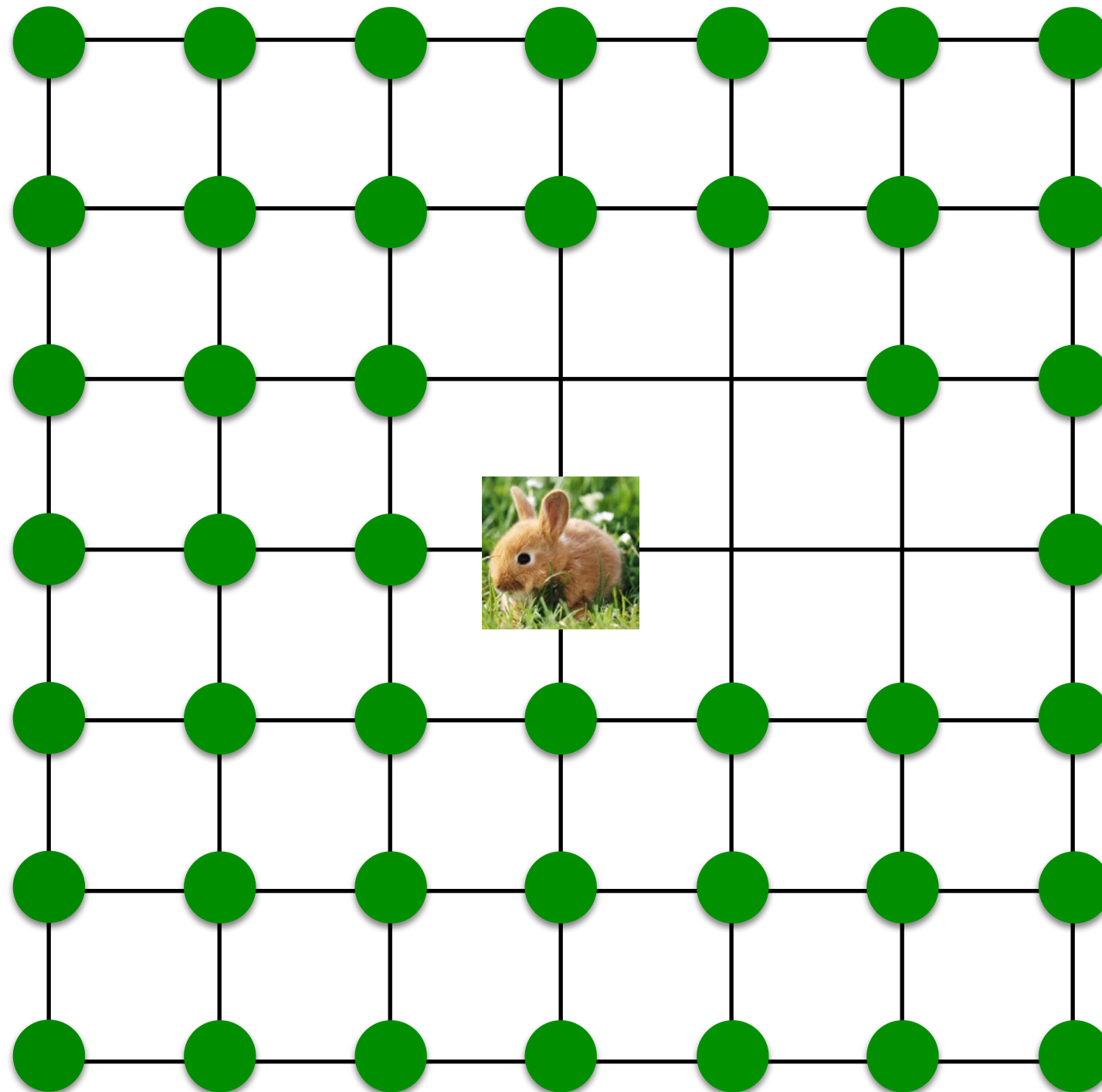
$$S = 6$$

time = 7

time left = 6

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



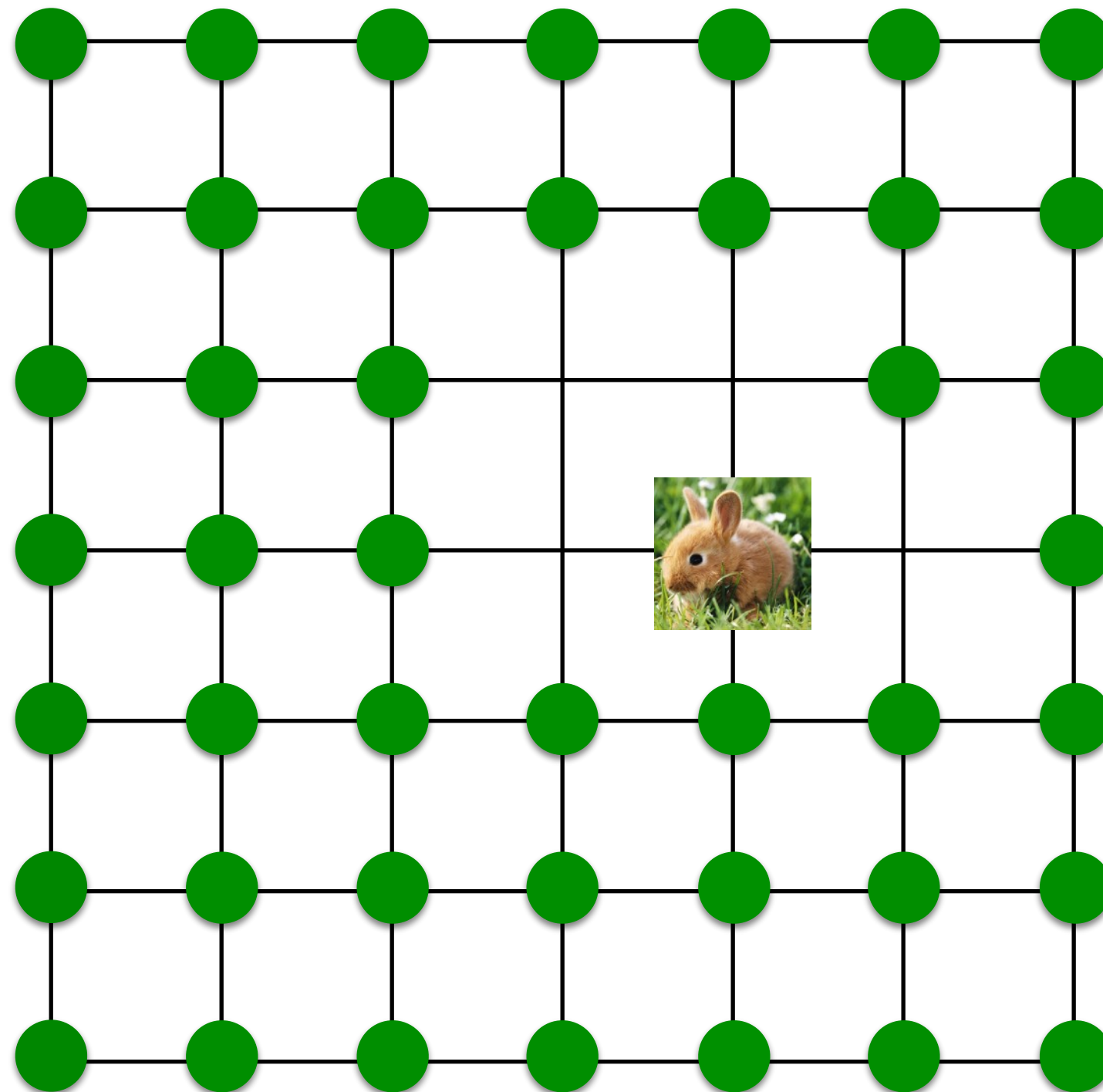
$$S = 6$$

time = 8

time left = 5

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



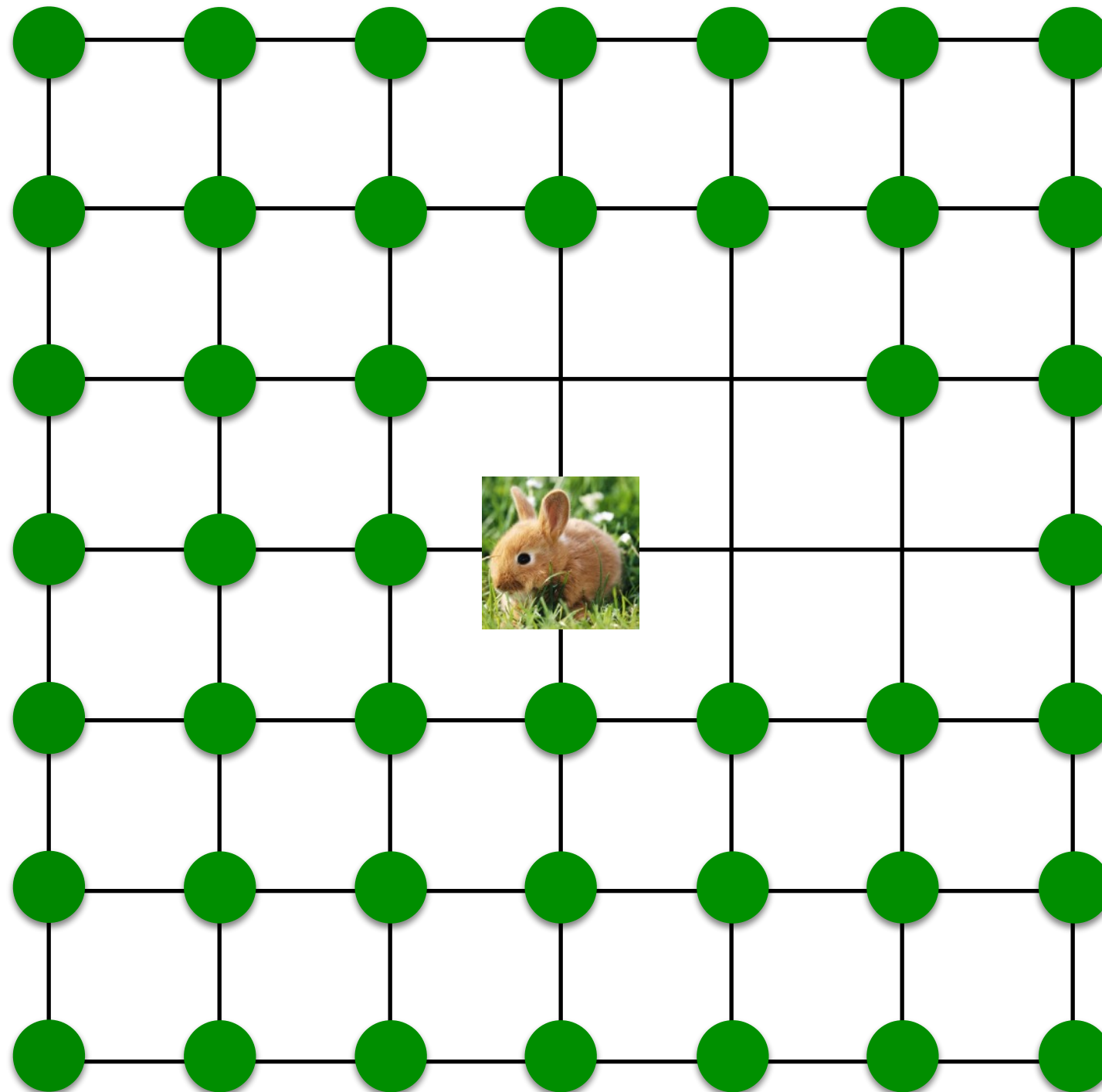
$$S = 6$$

time = 9

time left = 4

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



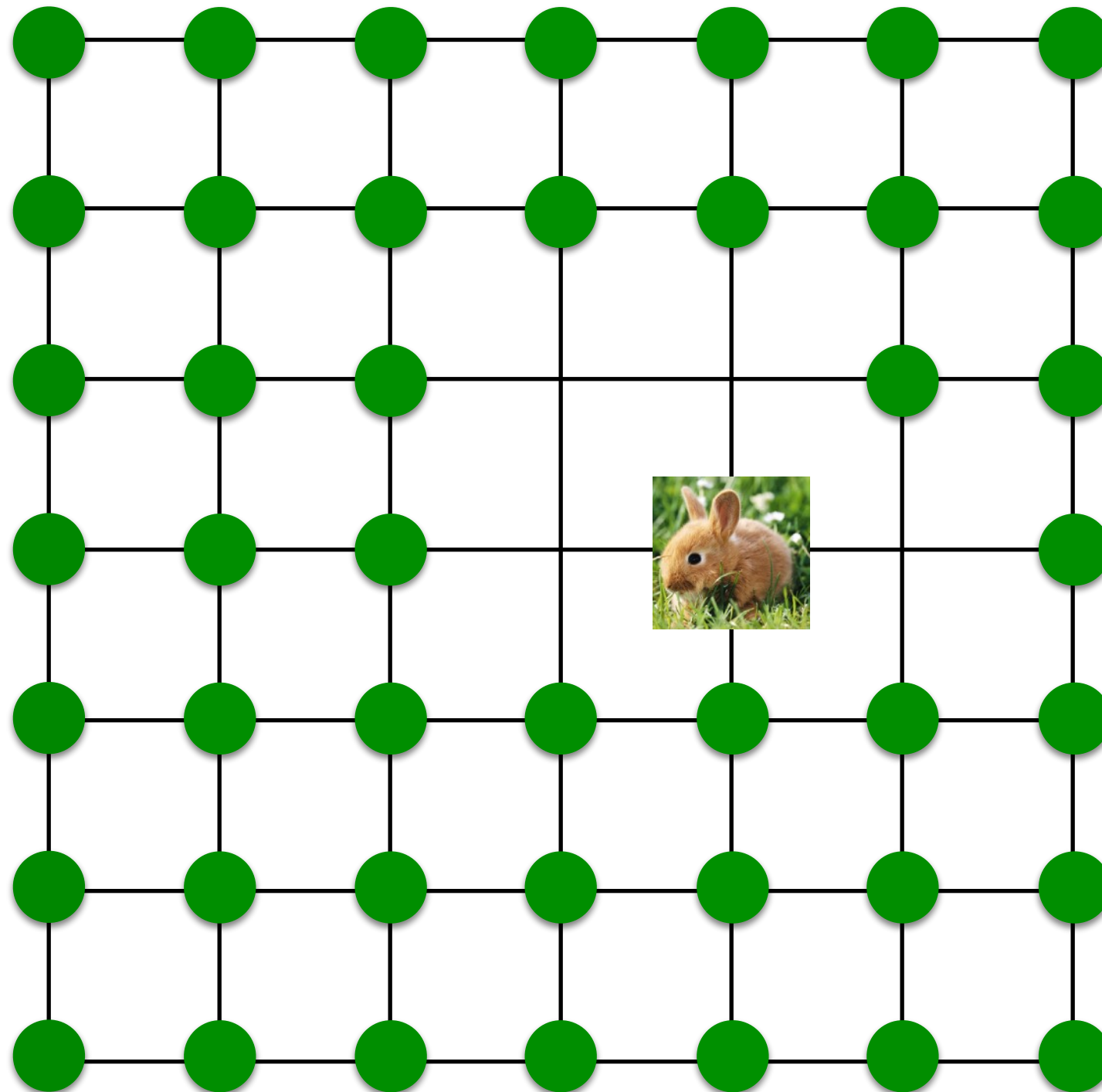
$$S = 6$$

time = 10

time left = 3

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



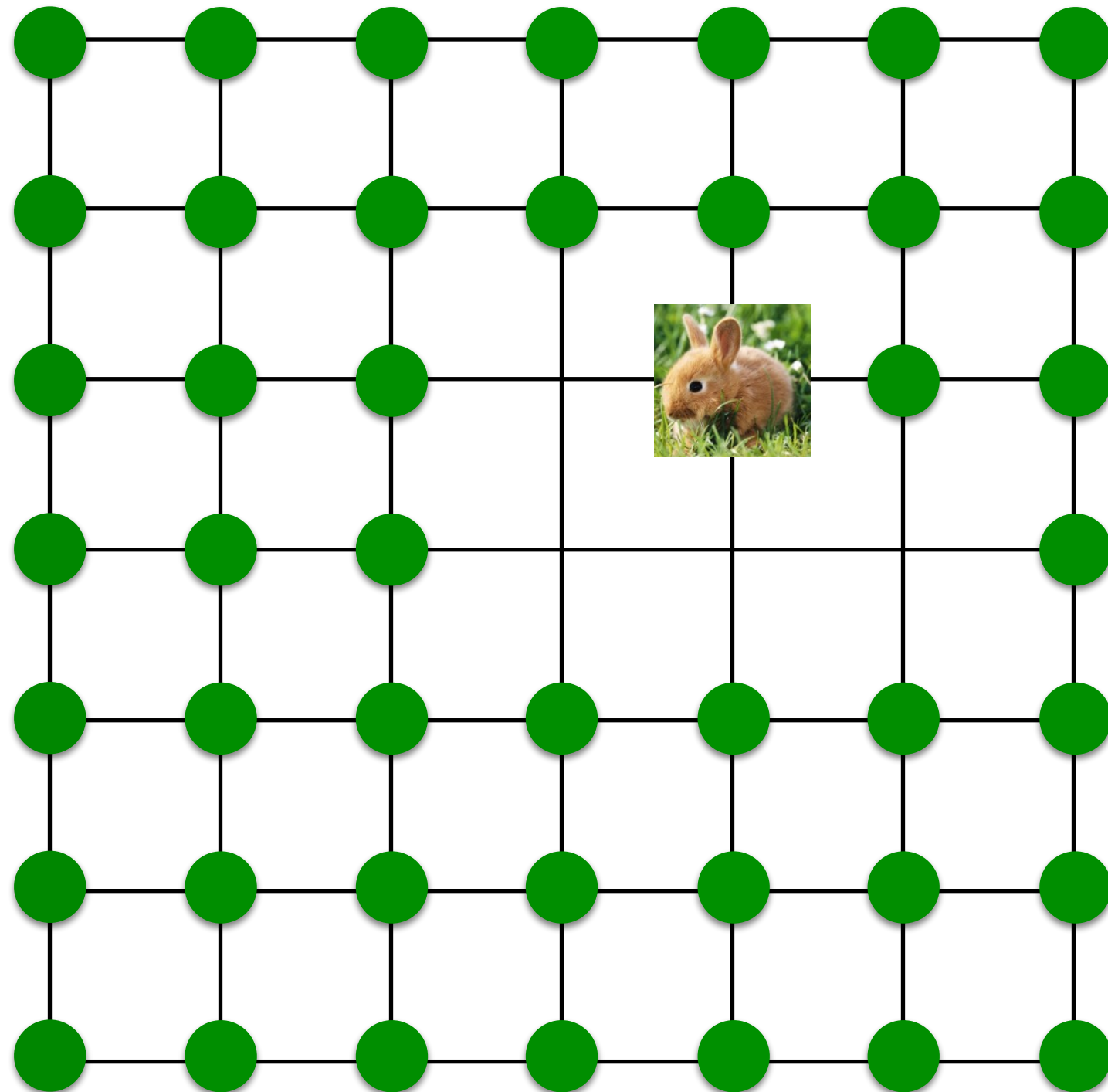
$$S = 6$$

time = 11

time left = 2

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



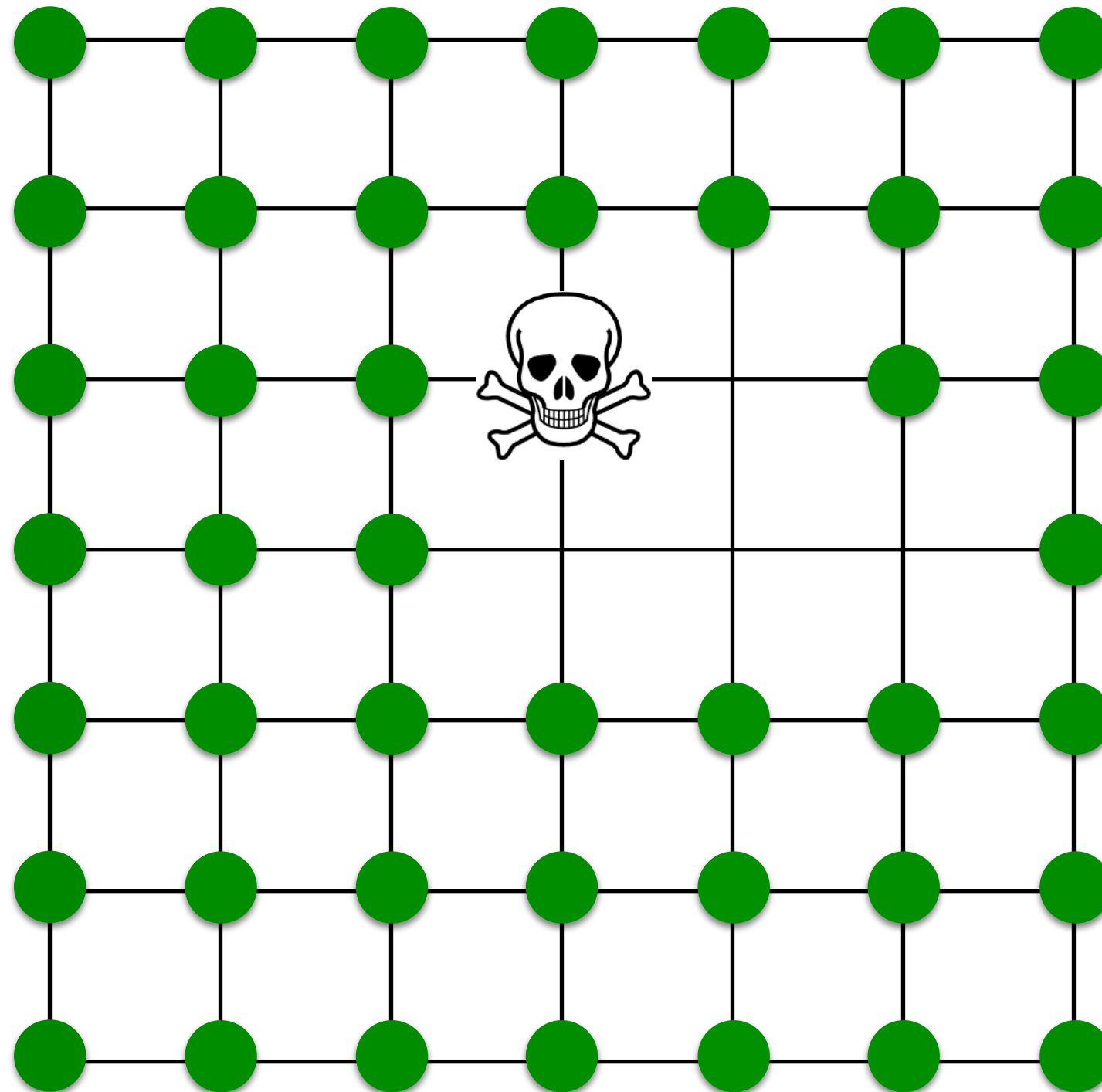
$$S = 6$$

time = 12

time left = 1

Starving Random Walk Model of Foraging

Fundamental parameter S , the stomach capacity



$$S = 6$$

time = 13

time left = 0

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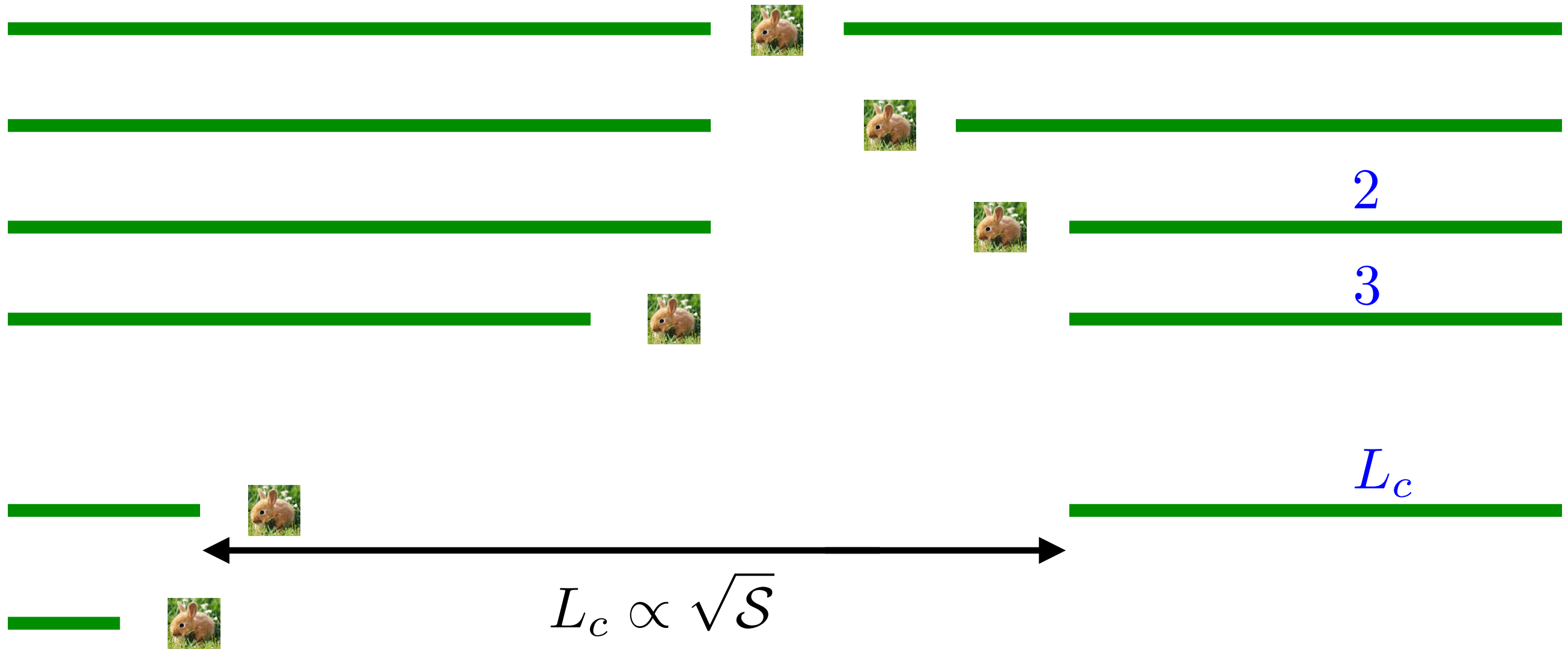
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Outline: Starving random walk model
dependence of lifetime on S & d

Greed is greed “good” or “bad”?

Outlook & open questions

Lifetime in One Dimension



$$t_{\text{build}} \simeq \frac{1}{2} L_c^2 \simeq S$$

Lifetime in One Dimension



$$R = \int_0^{\mathcal{S}} F(t) dt = \text{erfc}(1/\sqrt{4D\mathcal{S}}) \sim 1 - \mathcal{S}^{-1/2}$$

$$\# \text{ returns} \sim \mathcal{S}^{1/2}$$

$$\langle t \rangle = \int_0^{\mathcal{S}} t F(t) dt / \int_0^{\mathcal{S}} F(t) dt \sim \mathcal{S}^{1/2}$$

$$\mathcal{T} = t_{\text{build}} + (\# \text{ returns} \times \langle t \rangle) + \mathcal{S} \simeq 3\mathcal{S}$$

Lifetime in One Dimension

Lifetime: $\mathcal{T} = \sum_{n \geq 0} [\tau_1 + \tau_2 + \cdots + \tau_n + \mathcal{S}] V(n) \quad \tau_n = \frac{\int_0^{\mathcal{S}} dt \, t \, F_n(t)}{\int_0^{\mathcal{S}} dt \, F_n(t)} \simeq \int_0^{\mathcal{S}} dt \, t \, F_n(t)$

$$\begin{aligned} \int_0^{\mathcal{S}} dt \, t \, F_k(t) &= \frac{4\pi D}{(ka)^2} \sum_{j=0}^{\infty} (2j+1) \sin \frac{(2j+1)\pi}{k} \int_0^{\mathcal{S}} dt \, t \, e^{-\beta t} \\ &\simeq \frac{4\pi D}{(ka)^2} \frac{\pi}{k} \sum_{j \geq 0} (2j+1)^2 \left[\frac{k^2 a^2}{(2j+1)^2 \pi^2 D} \right]^2 [1 - e^{-\beta \mathcal{S}} (1 + \beta \mathcal{S})] \\ &\simeq \frac{4ka^2}{D\pi^2} \sum_{j \geq 0} \frac{1}{(2j+1)^2} [1 - e^{-\beta \mathcal{S}} (1 + \beta \mathcal{S})] \end{aligned} \quad \beta = [(2j+1)\pi]^2 D / (ka)^2$$

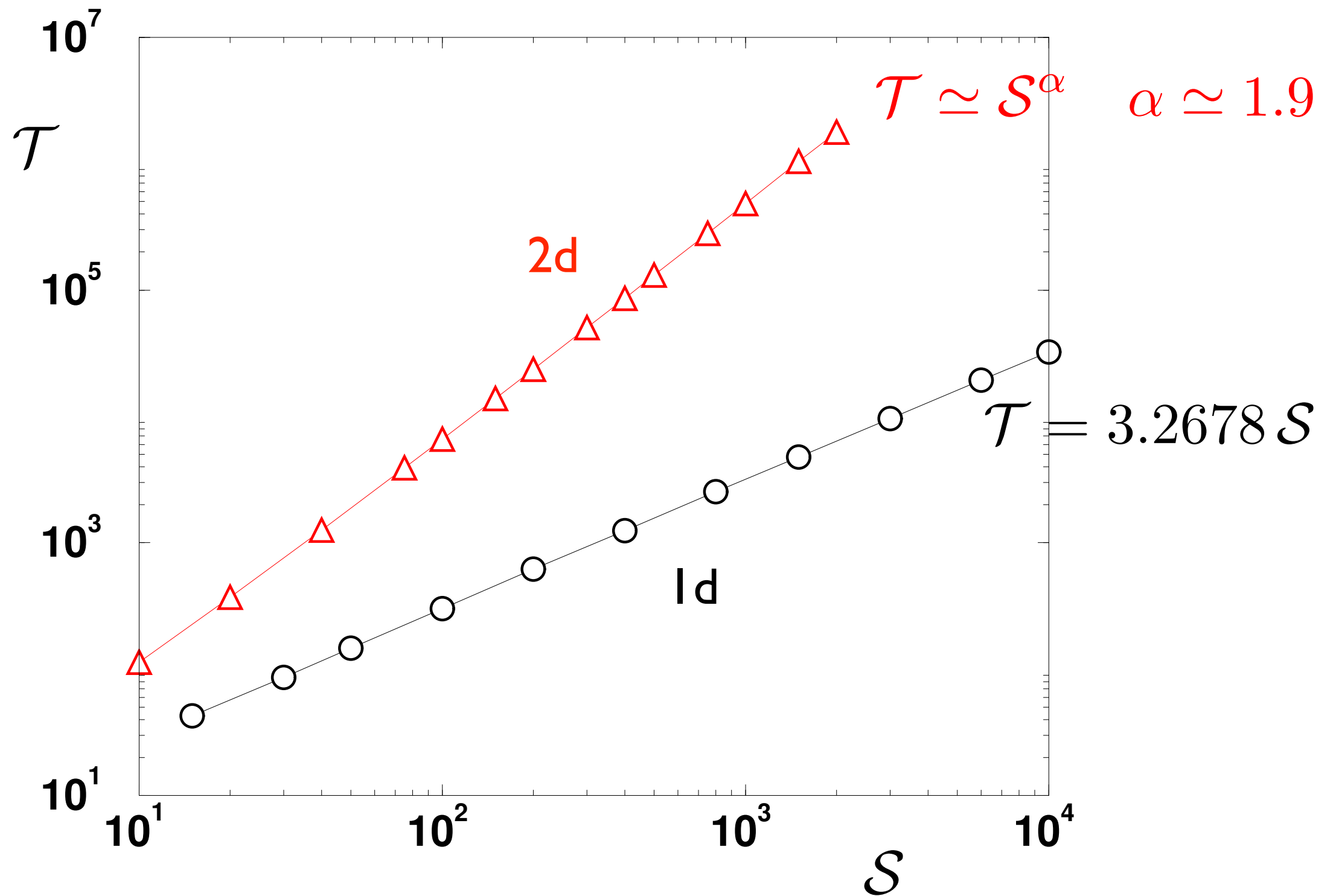
$$\sum_{1 \leq n \leq n} \tau_k \simeq \mathcal{S} \int_0^{na/(\pi\sqrt{D\mathcal{S}})} du \, u \sum_{j \geq 0} \frac{4}{(2j+1)^2} \left\{ 1 - e^{-(2j+1)^2/u^2} \left[1 + \left(\frac{2j+1}{u} \right)^2 \right] \right\} \quad u = ka/(\pi\sqrt{D\mathcal{S}})$$

Finally: $\mathcal{T} = \sum_{n=0}^{\infty} \left[\sum_{1 \leq k \leq n} \tau_k \right] V(n) + \mathcal{S}$

$$V(\theta) = \frac{4}{\theta} \exp \left\{ -2 \sum_{n \geq 0} E_1[(2n+1)^2/\theta^2] \right\} \sum_{n \geq 0} e^{-(2n+1)^2/\theta^2}$$

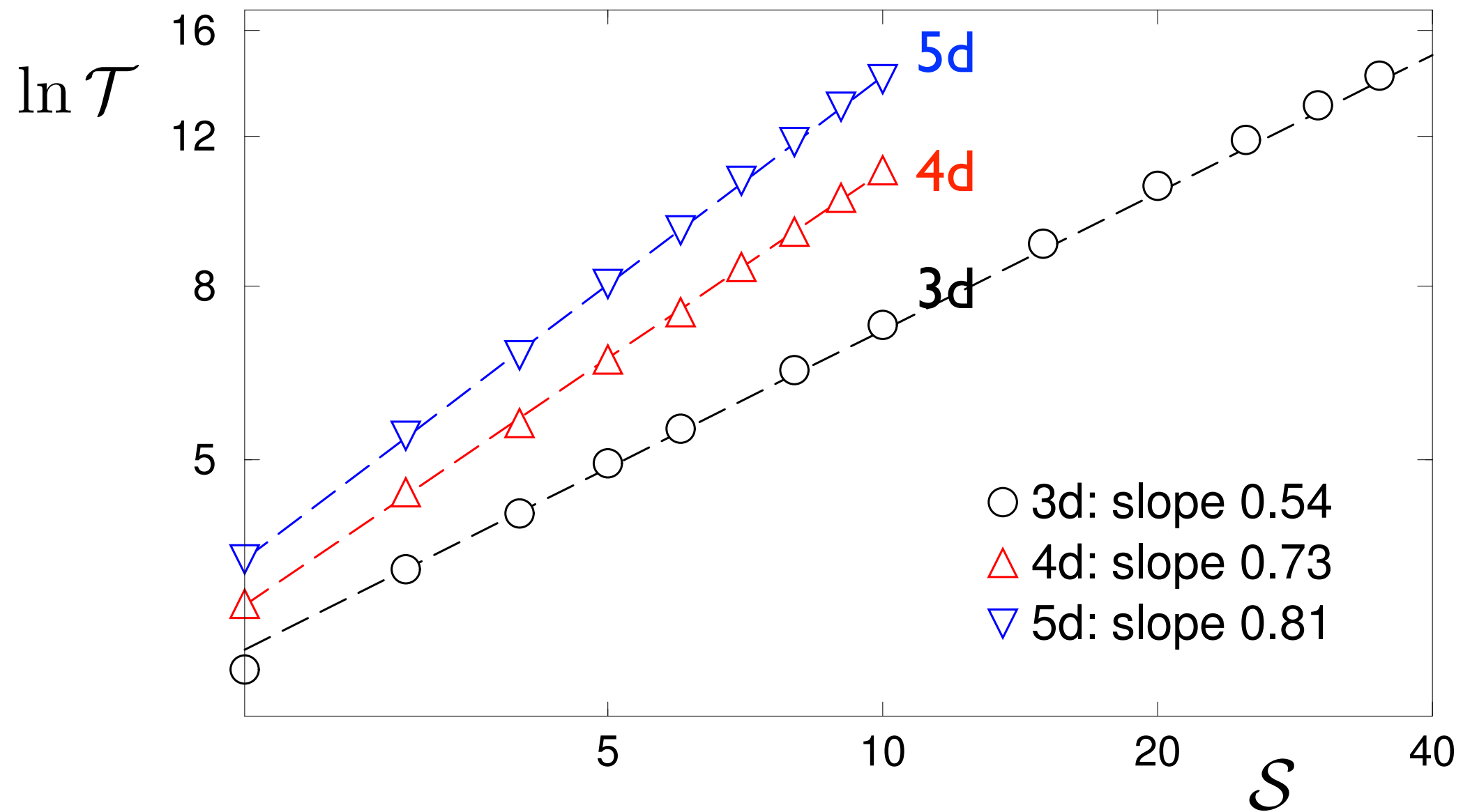
$$\simeq \mathcal{S} \int_0^{\infty} d\theta \, V(\theta) \int_0^{\theta} du \, u \sum_{j \geq 0} \frac{4}{(2j+1)^2} \left\{ 1 - e^{-(2j+1)^2/u^2} \left[1 + \left(\frac{2j+1}{u} \right)^2 \right] \right\} + \mathcal{S}$$

Lifetime in One and Two Dimensions



Lifetime in Higher Dimensions

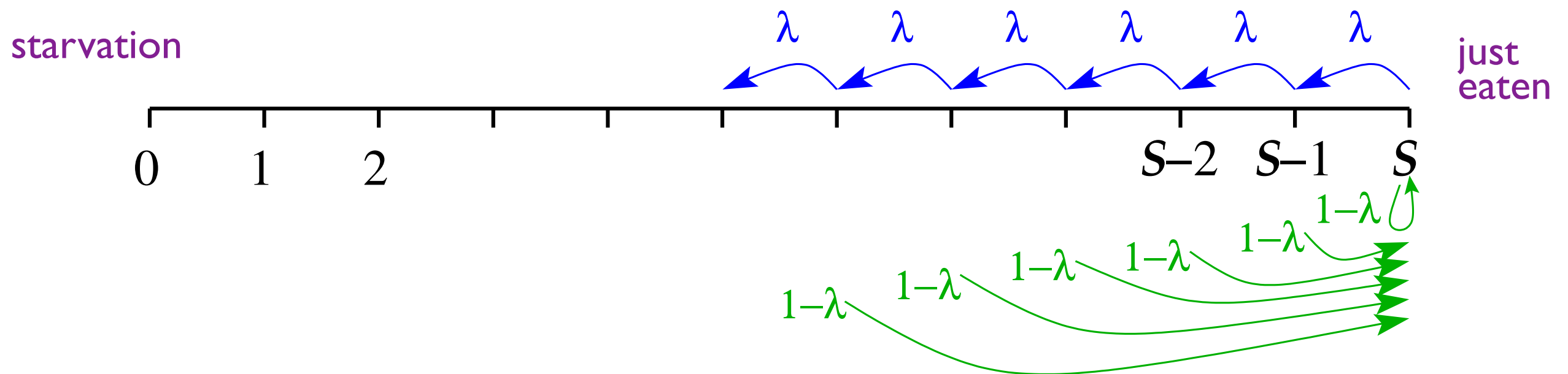
$$\mathcal{T} \sim \exp(\mathcal{S}^\omega)$$



$$\mathcal{T} \simeq \exp \left[\mathcal{S}^{(d-2)/d} \right] \quad \text{N. Smith}$$

Mean-Field Approximation

assume: site visitation governed by a Poisson process
 $\lambda \equiv$ prob. that RW hits a previously visited site



$t_n \equiv$ time to starve starting from site n

$$t_1 = 1 + (1 - \lambda)t_S$$

$$t_n = 1 + \lambda t_{n-1} + (1 - \lambda)t_S \quad 2 \leq n \leq S$$

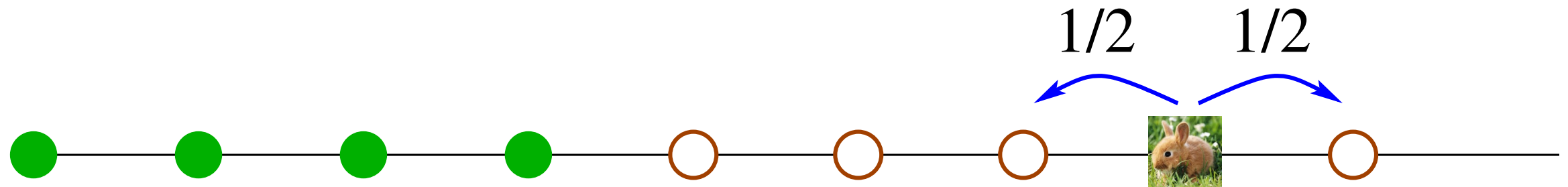
$$t_S \equiv \mathcal{T} = \frac{1}{\lambda^S} \left(\frac{1 - \lambda^S}{1 - \lambda} \right) \sim e^{-S \ln \lambda}$$

Role of Greed

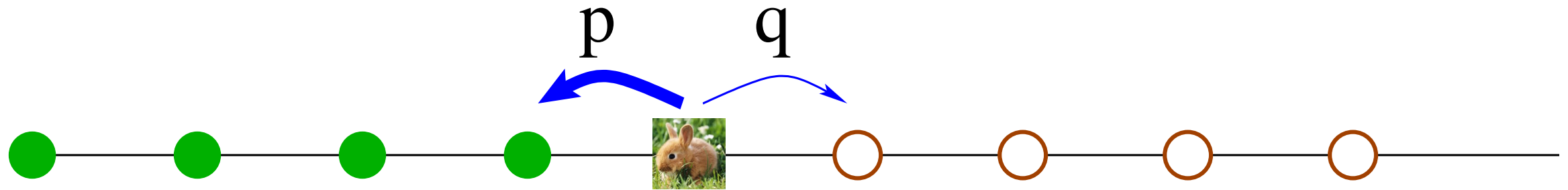
Greed is “good”: *consume resources as available*
→ *starvation unlikely*

Greed is “bad”: *enlarge desert more quickly*
→ *starvation more likely*

Greed in Id

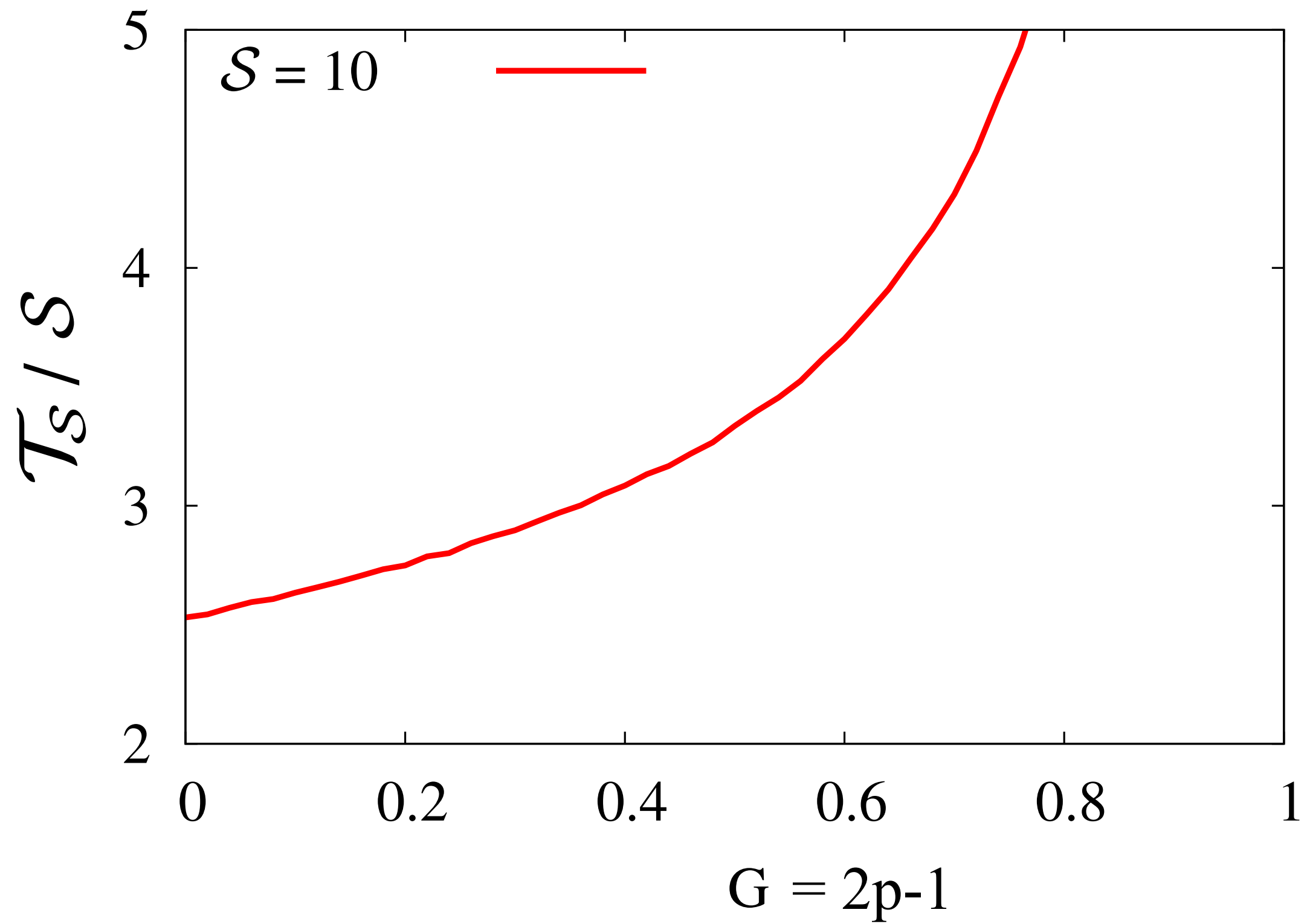


Greed in 1d

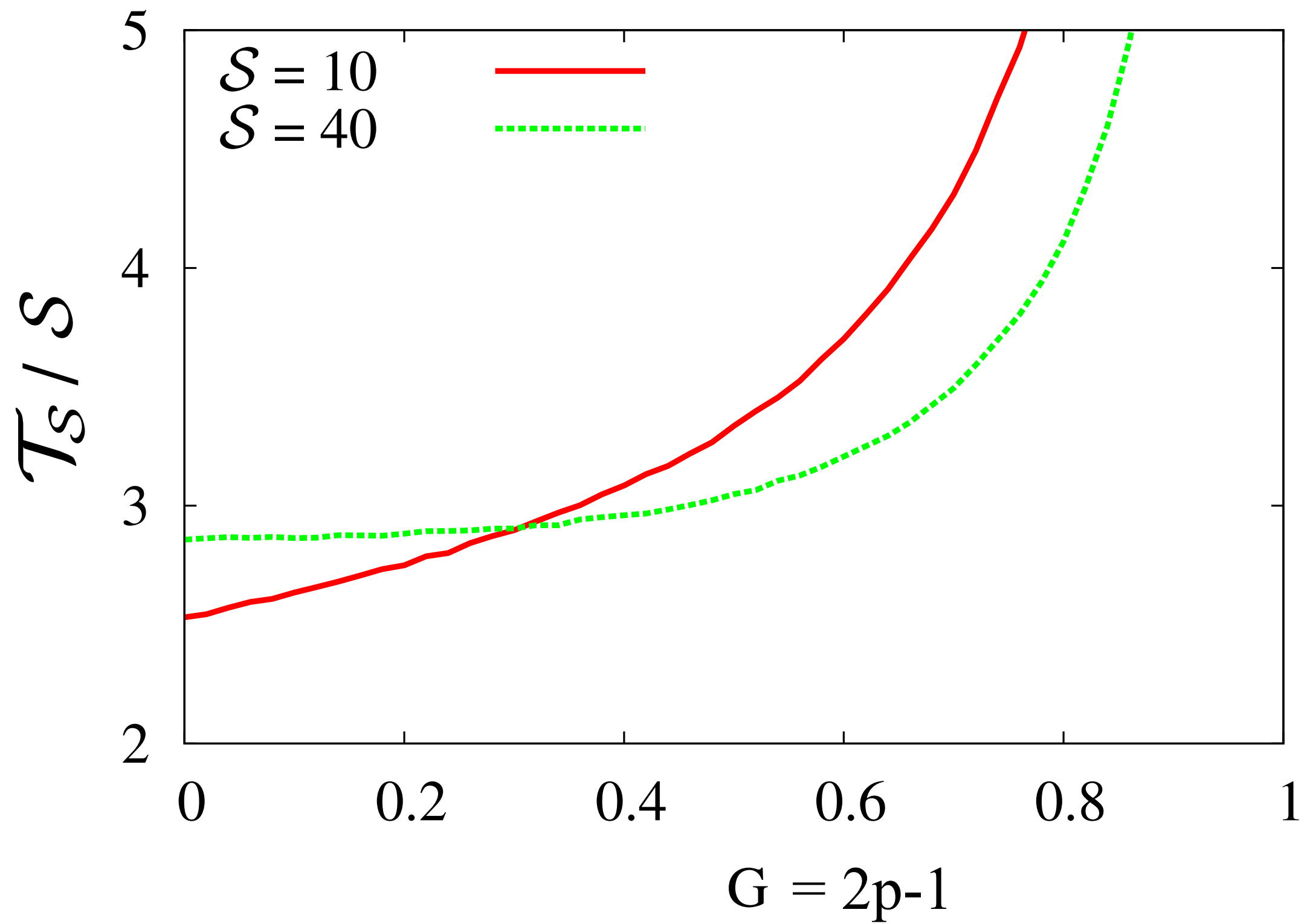


$$G=2p-1$$

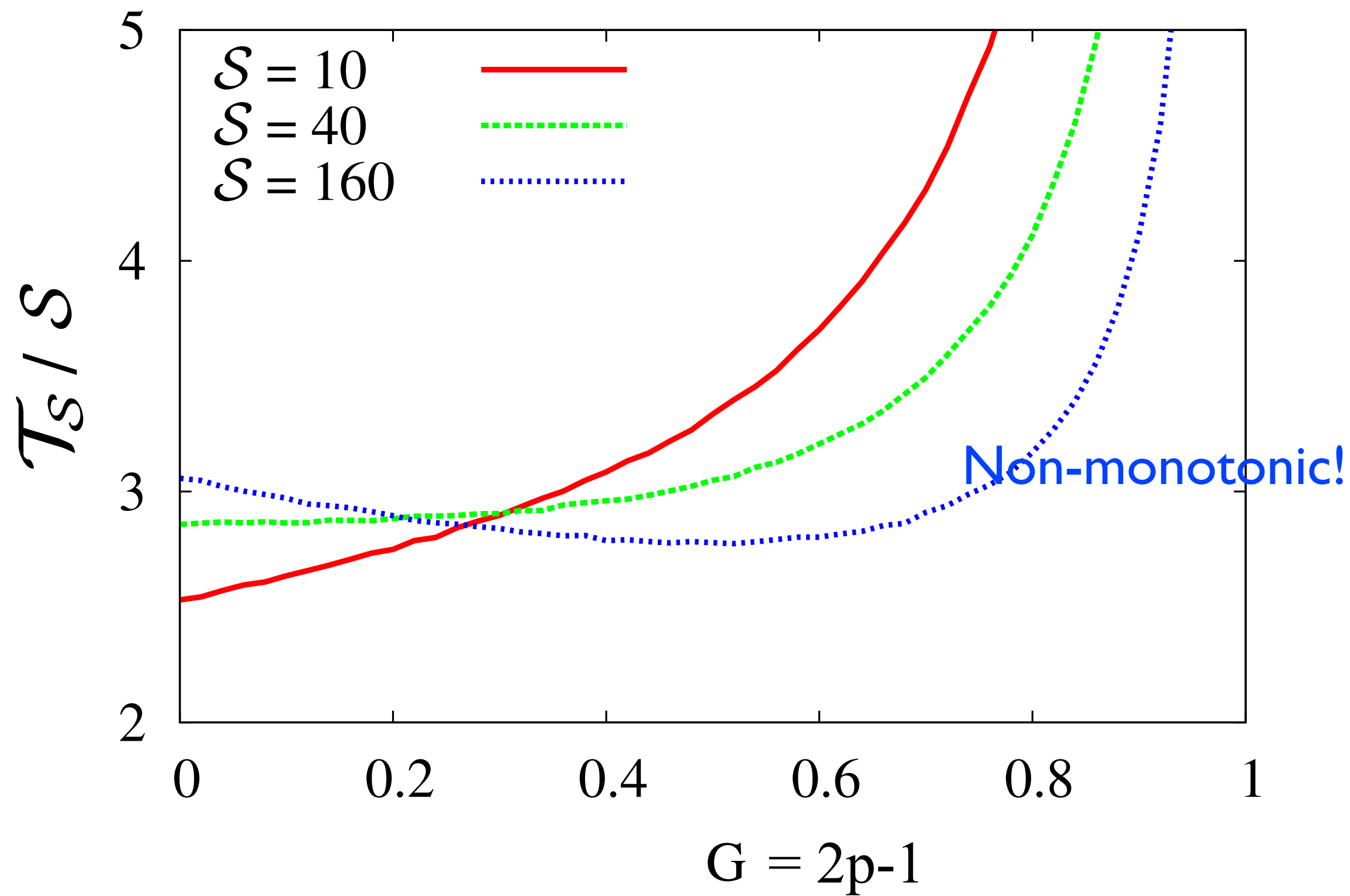
Greed in Id



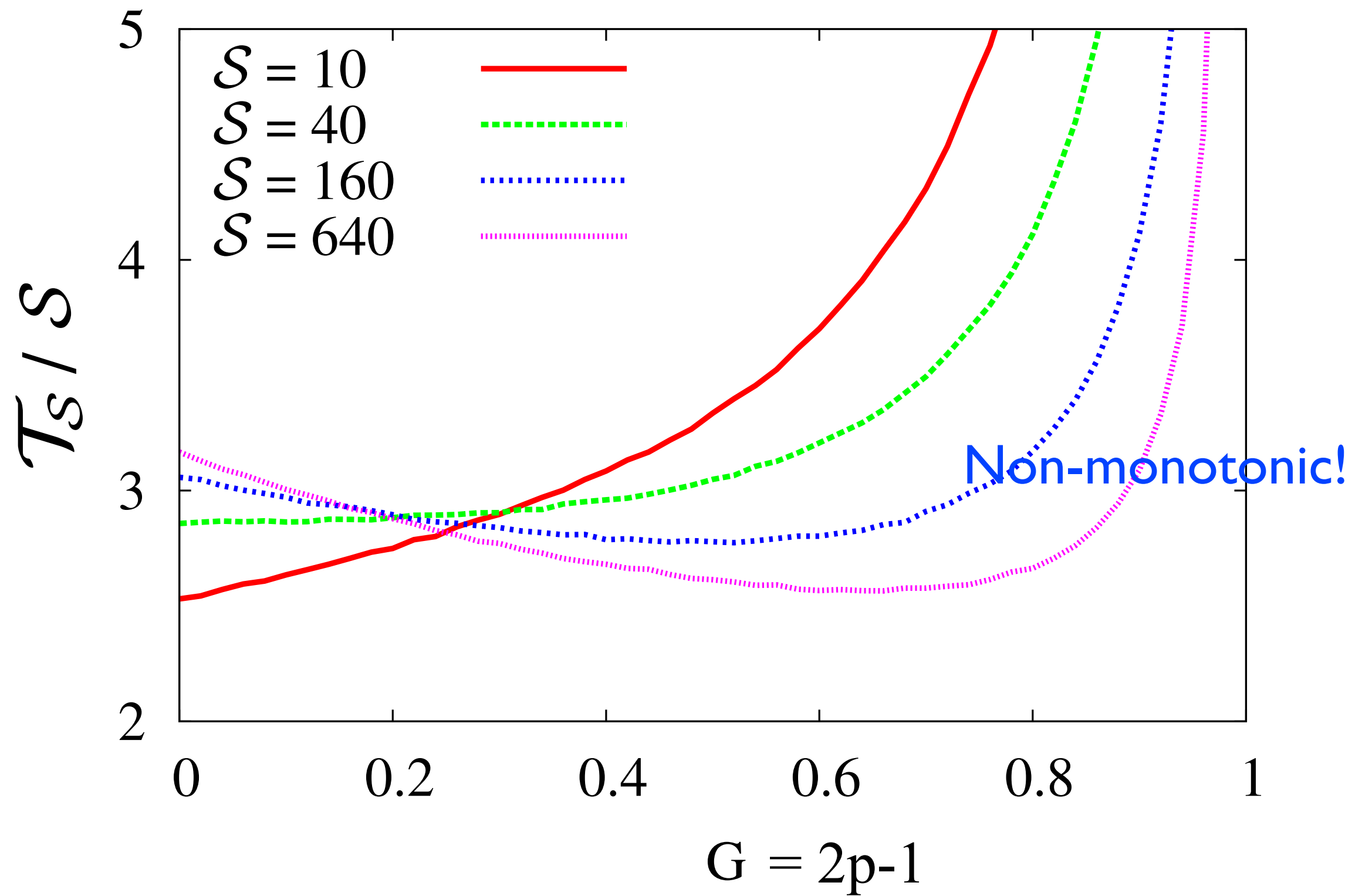
Greed in Id



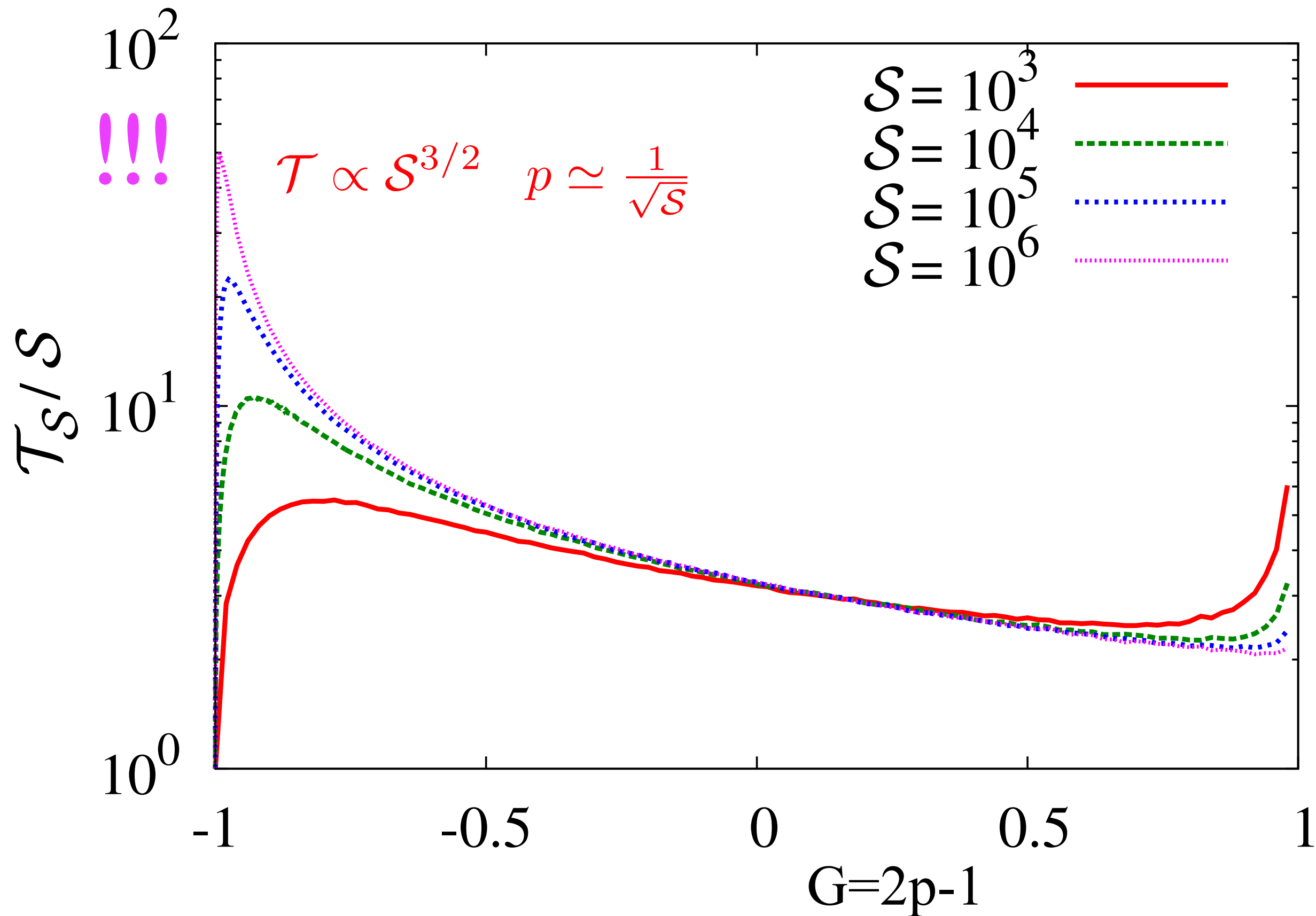
Greed in Id



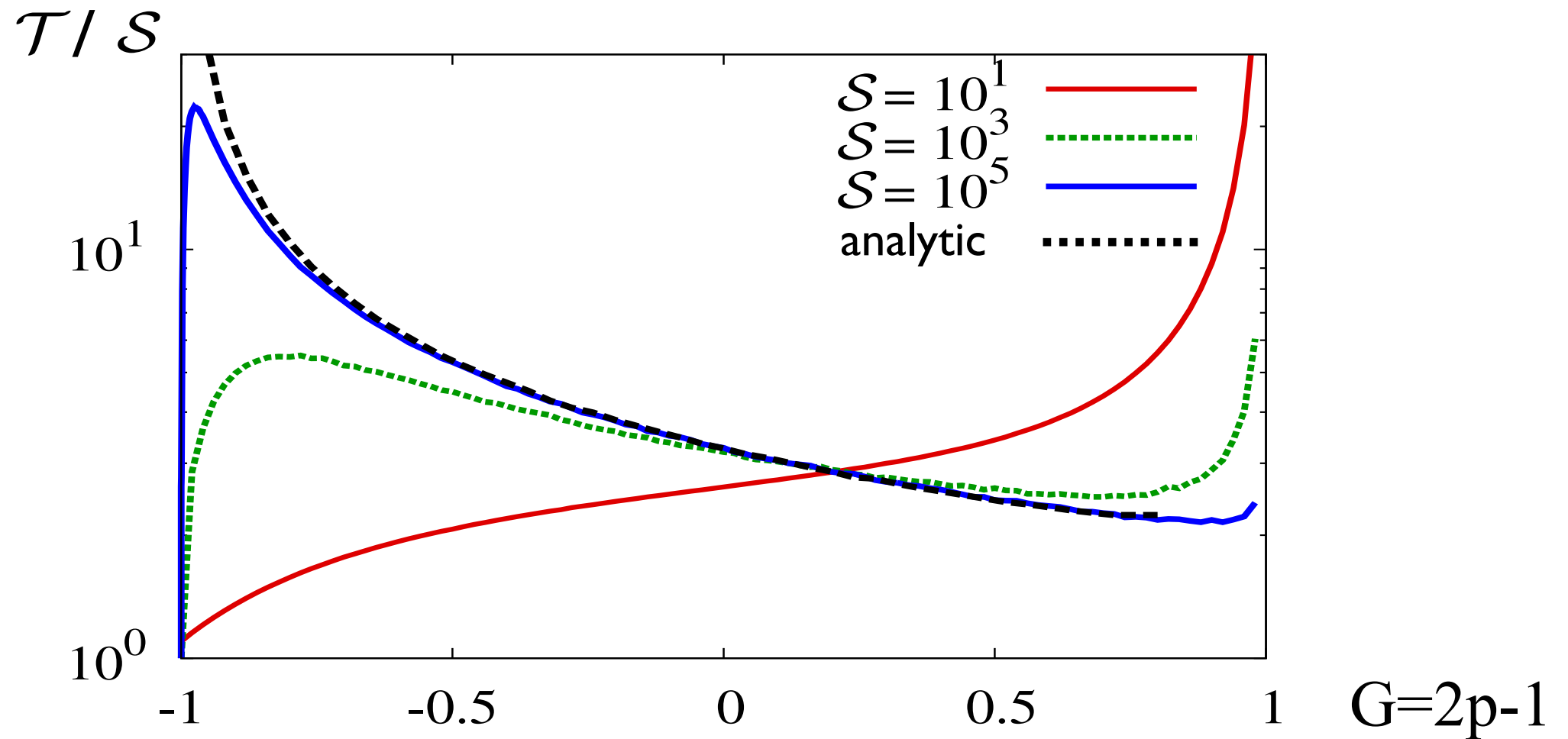
Greed in Id



Greed in Id



Greed in 1d

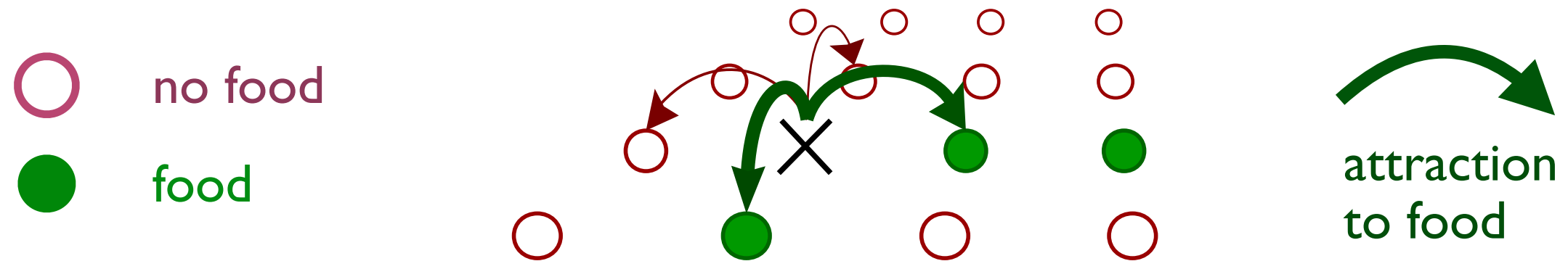


$$\mathcal{T} \simeq \mathcal{S} \frac{1-p}{p} \int_0^\infty d\theta V_\theta \int_0^\theta \frac{du}{u} \sum_{j \geq 0} \frac{4}{v^2} \left\{ 1 - e^{-v^2} [1 + v^2] \right\} + \mathcal{S}$$

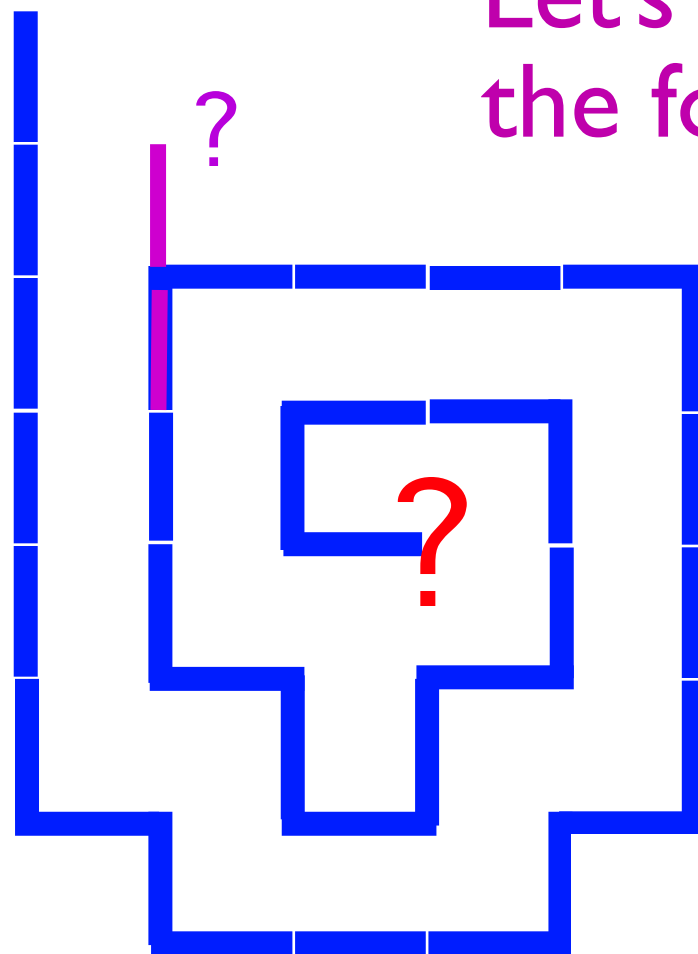
$$V_\theta \simeq \frac{4(1-p)}{p\theta} \sum_{j \geq 0} e^{-w^2 - Q} \quad Q = \frac{2(1-p)}{p} \sum_{j \geq 0} E_1(w^2)$$

$$E_1(x) = \int_1^\infty dt e^{-xt} / t \quad v = w = (2j+1)/u \quad \theta = n / (\pi \sqrt{D\mathcal{S}})$$

Greed in 2d



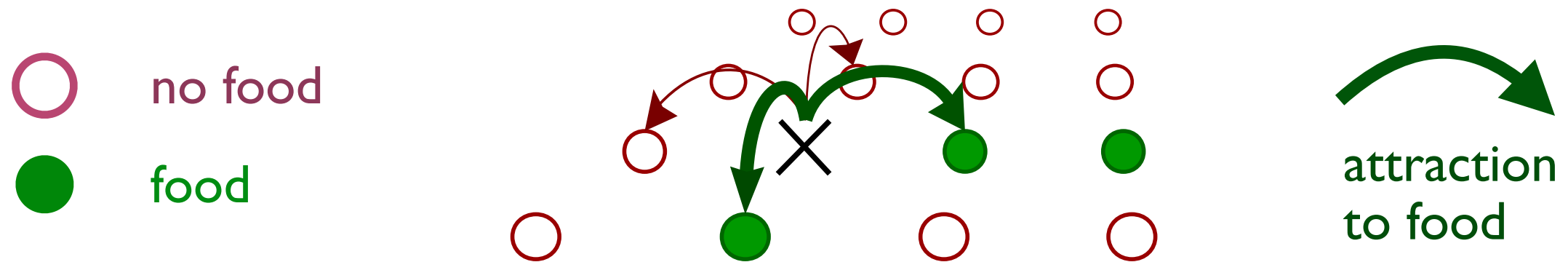
Let's see what can happen when the forager is really, really greedy



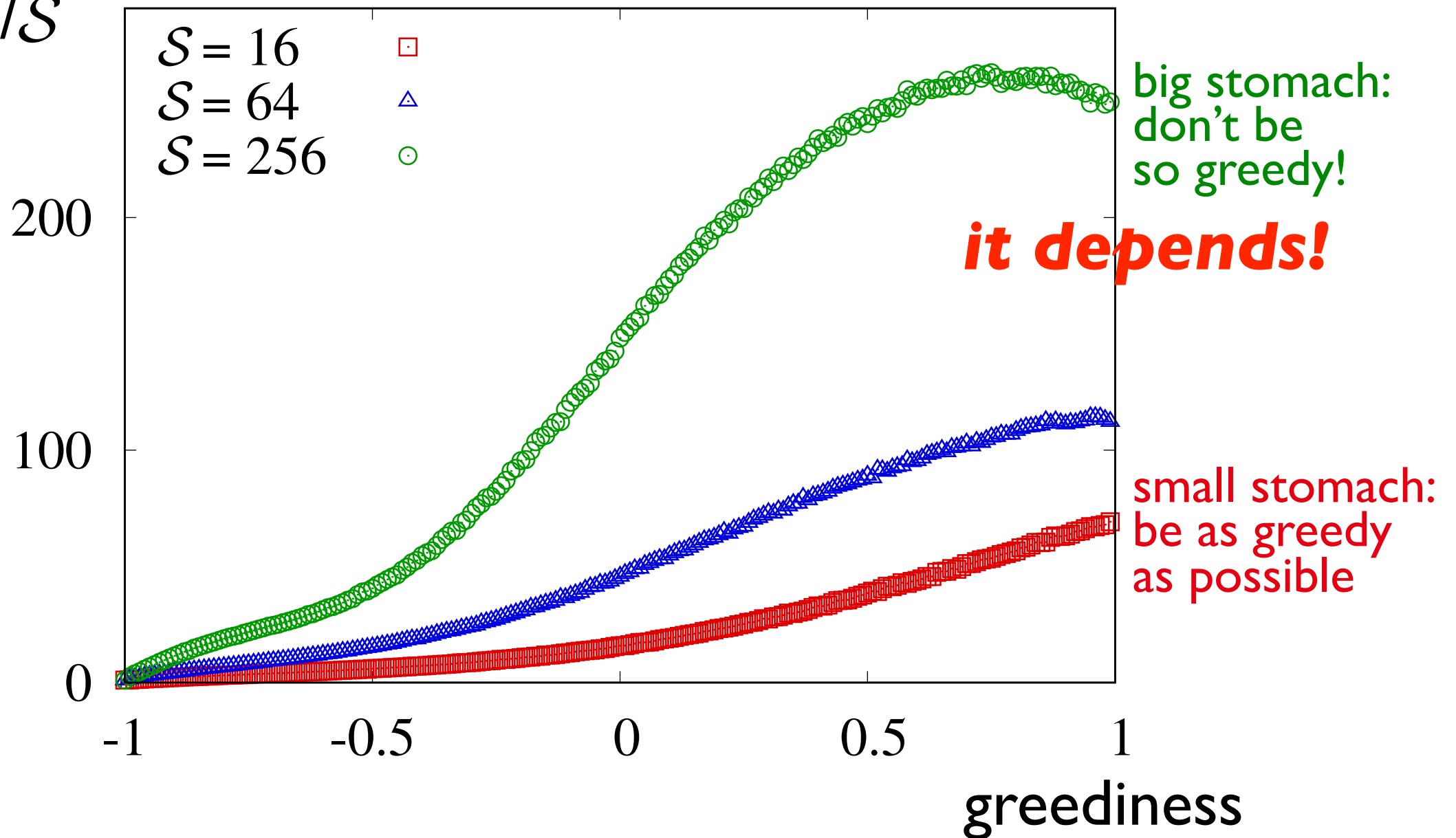
Now what?

Starvation likely!

Is Greed Good or Bad in 2d?



lifetime/ $\mathcal{S}$



Outlook & Open Questions

- Dependence of lifetime on S and dimension ✓—
- Is greed good or bad? ***It depends!***
- Is it good to be clueless?
eat with probability p ***Yes!***
- Is it good to be frugal?
only eat $\leq k$ steps before starvation ***Yes!***
- Resources replenish? ***Immortality transition***
- Patchy resources ***Explore vs. exploit criterion***
- Birth coupled to nutrition & allometry ***optimal body size***
J. Yeakel, C. Kempes
- Multiple foragers ???
- Predation ???
- No overfilling ???

Will you make it to
the next food
break?